

# Credit Spreads with Dynamic Debt <sup>1</sup>

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## Abstract

### Credit Spreads with Dynamic Debt

This paper extends the baseline Merton (1974) structural default model, which is intended for *static* debt spreads, to a setting with *dynamic* debt, where leverage can be ratcheted up as well as written down through pre-specified exogenous policies. We provide a different and novel solution approach to dynamic debt than in the extant literature. For many dynamic debt covenants, ex-ante credit spread term structures can be derived in closed-form using modified barrier option mathematics, whereby debt spreads can be expressed using combinations of single barrier options (both knock-in and knock-out), double barrier options, double-touch barrier options, in-out barrier options, and one-touch double barrier binary options. We observe that debt principal swap down covenants decrease the magnitude of credit spreads but increase the slope of the credit curve, transforming downward sloping curves into upward sloping ones. On the other hand, ratchet covenants increase the magnitude of ex-ante spreads without dramatically altering the slope of the credit curve. These covenants may be optimized by appropriately setting restructuring boundaries, which entails a trade-off between the reduction in spreads against restructuring costs. Overall, explicitly modeling this latent option to alter debt leads to term structures of credit spreads that are more consistent with observed empirics.

*Keywords:* Credit spreads; dynamic debt; ratchet; restructure; guarantee; barrier options.

# 1 Introduction

Predicting and pricing the likelihood of default is important to investors, lenders, and debtors alike, and accordingly, a substantial body of work attempts to model and price risky debt claims, and to determine related credit spreads. Beginning with Black and Scholes (1973) and Merton (1974), standard structural models start with a riskless claim, subtracting out the value of a guarantee on a *fixed* debt level, which represents the value of the borrower’s option to default. Empirically, however, firms that issue debt, actively manage their debt structure and levels, and debt rarely remains fixed. This paper models in closed form, using barrier options, the magnitude of and changes to ex-ante spreads when accounting for the fact that debt is dynamically updated under flexible rules. This analytic and parsimonious extension of the Merton model generates spread curves for high-yield debt that match the shapes observed in practice.

In the classic Merton (1974) framework with *static* zero-coupon debt, the risky debt discount is priced by a plain vanilla put option on the underlying firm with a strike equal to the current debt principal, the value of which can be translated into credit spreads on the firm’s debt. To this model, we add features that allow debt to be ratcheted up or written down. That is, we allow for a possible increase in a firm’s debt level (i.e., a *ratchet*) in response to increases in underlying firm value; we also allow for a possible decrease in its debt level (i.e., a *swap down*) that replaces debt principal with equity in response to decreases in underlying firm value, a process also referred to as “de-leveraging”.<sup>1</sup>

Specifically, we show that extensions of the static debt Merton model to debt discounts for credit risk (and hence spreads) on *dynamic* debt can be derived analytically using barrier options, a class of exotic derivatives that are activated or de-activated upon accessing a pre-determined barrier. This paper provides a range of solutions for spreads on dynamic debt using different barrier option types, such as single barrier options (both knock-in and knock-out), double barrier options, double-touch barrier options, in-out barrier options, and one-touch double barrier binary options.

Intuitively, if the underlying assets increase sufficiently in value, then the firm can use the

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<sup>1</sup>We use the term “swap down” instead of debt write down because we specifically mean a replacement of debt with equity, and not a mere write down of debt. Correspondingly, when debt is “ratcheted” up we reduce the amount of equity by an equal amount.

extra collateral to support more debt, thereby *ratcheting* the debt to firm value ratio upward and increasing the debt discount. That is, once the underlying firm value appreciates to an upper barrier, the original put option on debt is knocked out and replaced by another put at a higher strike representing the increased level of debt. Thus, in contrast to the plain vanilla put representing the Merton discount on non-renegotiable debt, the value of a discount on debt with the option to ratchet is decomposed into two single-barrier options: an up-and-out put option to capture the discount on the original level of debt, and an up-and-in put option to capture the new discount at the increased debt level.

Analogously, *swap downs* may occur when the underlying assets decrease substantially in value, and the put option to default becomes deep in-the-money. To stave off default, lenders can swap debt principal for equity to make the default option less profitable to exercise from the borrower's standpoint.<sup>2</sup> Thus, the value of a discount on debt that may subsequently be reduced can be expressed as the sum of two single-barrier options: a down-and-out put option struck at the original debt level, and a down-and-in put option struck at the reduced debt level.

Under this framework, we obtain closed-form solutions for the ex-ante value of the debt discount and corresponding credit spread term structure, explicitly modeling the latent option to either ratchet or swap down debt after issuance. We also extend this pricing model to allow for various combinations of possible ratchets and swap downs. Although the resulting barrier-option representation of the debt discount in such a setting is much more complex than in the single ratchet or single swap down cases, the solutions are analytical and lead to intuitive and empirically known shapes of the term structures of credit spreads. These results may also be extended recursively to more complicated repetitive opportunities to alter debt.

Overall, this parsimonious extension of the static debt structural model in closed-form using barrier options results in more empirically tenable term structures of credit spreads. The main results of our analyses are as follows:

1. *Level effect:* (a) Debt discounts and credit spreads increase with ratchets and decrease with principal swap down features. (b) The ratchet effect is more pronounced for medium-debt

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<sup>2</sup>This is now a prevalent practice in the mortgage markets, supported by government regulation (e.g., see the HAMP-PRA scheme). A recent example in the case of sovereign debt is the forgiveness of principal on Greek debt.

firms than for high-debt firms, because ratchets occur at lower leverage. Similarly, the swap down effect is more pronounced for high-debt firms (than for medium-debt firms).

2. *Slope effect:* For high-debt firms, accounting for the swap down feature removes the downward bias in the slope of the yield curve, matching empirical evidence presented by Helwege and Turner (1999) and Huang and Zhang (2008).
3. *Optimal covenants:* (a) Covenants that restructure debt at a pre-specified market leverage (debt-to-value) ratio reduce ex-ante spreads, and these boundaries may be optimally chosen to trade off benefits of spread reduction against costs of frequent restructuring. (b) As the restructuring leverage level is reduced, spreads drop rapidly at first and then slowly; set against restructuring costs that are convex in restructuring likelihood and frequency, implies an optimal restructuring barrier.

Ours is not the first paper to extend the classic Merton (1974) structural model for risky debt.<sup>3</sup> However, credit spreads and curves predicted by these other models do not adequately match empirical observations of actual spreads and curves, as evidenced in Eom, Helwege, and Huang (2004), who empirically test five different structural models for corporate spreads. Although the Merton model produces spreads that are too low, these newer models produce spreads that are generally far too high. For example, the Longstaff and Schwartz (1995), Leland and Toft (1996), and Collin-Dufresne and Goldstein (2001) models predict spreads that are oftentimes more than double the actual spread. We depart from these studies in the following ways.

First, in contrast to these models, we use barrier options to explicitly model the option to ratchet or de-leverage, whereby the option to alter debt is exercised discretely upon accessing a

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<sup>3</sup>Other studies departing from this traditional paradigm include Longstaff and Schwartz (1995), who extend the structural class of models to default with the additional feature of stochastic interest rates; Leland (1994) and Leland and Toft (1996), who consider credit spread term structures under the choice of optimal capital structure and debt maturity with taxes and an endogenous bankruptcy barrier; Goldstein, Ju, and Leland (2001), who allow for possible increases in future debt levels; and Collin-Dufresne and Goldstein (2001), (CDG), who examine credit spreads under a mean-reverting capital structure in a setting where leverage is a stochastic process continuously tracking a pre-determined target. Our paper differs from CDG in the following ways. First, the debt level (default barrier) in CDG is continuously changing, whereas ours ratchets or swaps down only when barriers are breached. These punctuated changes in leverage are more natural. Second, mean-reverting leverage models assume both increases and decreases in leverage as mean-reversion occurs, and are less flexible than a model in which debt levels may increase or decrease separately, providing more varied features to the spread term structure. In our model we allow separate handling of increases and decreases in debt, with the same number of parameters as in CDG.

threshold and the debt level does not undergo continuous changes. In practice, debt levels do not change continuously as modeled by Collin-Dufresne and Goldstein (2001), and in modeling discrete, periodic, firm value-dependent revisions in debt levels, we observe substantive differences in predicted credit spreads and curves.

Second, structural models based on mean-reverting models of leverage do not place explicit bounds on the levels of debt the firm might carry, though by increasing the rate of mean reversion, the expected range in which the leverage lies can be controlled. In these models, sufficiently high speeds of adjustment are necessary to generate the upward sloping credit curves empirically observed on high-yield debt. But paradoxically, imposing high speeds of mean reversion results in leverage itself being less dynamic, and firms do not usually evidence such strict adherence to a target capital structure. In contrast, our “leverage barrier” model permits free movement of leverage within the pre-specified barriers and generates mean reverting capital structures with dynamic and periodic debt adjustments, concomitant with actual practice and consistent with the literature on bounded capital structures arising from costly readjustment, as modeled in Fischer, Heinkel, and Zechner (1989a).

Third, the extant literature has focused on different trade-offs than the one we consider here. Leland (1994) and Leland and Toft (1996) model firm-value optimizing debt policies trading off tax shields and deadweight bankruptcy costs. These debt policies are endogenous, and are more apt when considering policy making by a firm. In contrast to these papers, we minimize the cost of debt funding (spreads) by trading off the cost of restructuring versus the reduction in spreads from covenants that impose de-leveraging. The restructuring boundaries in our paper are exogenous, making the model simpler to implement, and more apt for use by investors, who might use credit spreads to infer implied restructuring boundaries, or firms, who choose ex-ante restructuring boundaries to manage their credit spreads. Hence, we depart from traditional optimal capital structure based models of dynamic debt choice to a model with simpler closed-form barrier option-based solutions that easily match observed empirical characteristics of yield curves. Our exogenous barriers model is a direct extension of Merton (1974) that does not require specific trade-offs of tax shields and bankruptcy costs.

Finally, under this framework, we obtain credit spreads and curves that more closely match prior

empirical observations, not only in the shape of the curve but also in the magnitude of the spreads. Moreover, our main features, which we have enumerated above, match model prescriptions based on recent empirical tests of existing pricing models; in particular, that “more accurate structural models must avoid features that increase the credit risk on the riskier bonds while scarcely affecting the spreads of the safest bonds” (Eom, Helwege, and Huang, 2004). We chose to extend the static debt model to a dynamic one in the simplest framework possible, i.e., the Merton (1974) model, rather than more complex versions of structural models such as Longstaff and Schwartz (1995), Leland and Toft (1996). This ensures that there are no other model features other than the transition from static to dynamic debt that result in much better matching of empirical regularities of the credit spread term structure.

Specifically, traditional structural models (on fixed debt) predict upward sloping yield curves on higher quality debt, but downward sloping yield curves on very risky debt. However, empirical evidence suggests that yield curves are mostly upward sloping, and that this pattern applies to both coupon-paying and zero-coupon debt of varying credit qualities (Huang and Zhang, 2008), including high-yield coupon bonds (Helwege and Turner, 1999).<sup>4</sup> Thus, traditional structural models do not capture empirically observed features of credit spread curves for corporate bonds.<sup>5</sup> In contrast, our model does.

That our model matches the empirical features of credit spread curves comes from the fact that debt levels are altered in precisely the way we have modeled them, i.e., changes to a firm’s debt levels over time may arise as the firm decides to issue new debt or to recall existing debt at threshold levels,<sup>6</sup> while wandering without forced direction between these levels. A firm’s managers may set such optimal debt boundaries to manage ex-ante spreads to a level that is acceptable to

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<sup>4</sup>In contrast, the evidence in Lando and Mortensen (2005) from CDS markets finds downward sloping credit spread curves even for high-debt firms, and Han and Zhou (2010) find upward sloping curves except for the worst quality firms; however, CDS credit curve slopes are viewed between one and five years and are very different in maturity than bond credit curves.

<sup>5</sup>Analytic evidence suggests that, in contrast to structural models, reduced-form models may be more adept at matching credit term structures. See for example Madan and Unal (2000).

<sup>6</sup>Debt may also change through negotiated decisions with existing debt holders to alter major contract terms, such as the principal amount, maturity, and associated debt covenants. Empirical evidence suggests that value-enhancing restructurings take place in cases of actual as well as technical default (Nini, Smith, and Sufi, 2012). Furthermore, evidence suggests that debt is renegotiated even in the absence of financial distress, as new information is realized about the firm’s prospects and credit quality (Roberts and Sufi, 2009a,b; Garleanu and Zwiebel, 2009).

debt holders. Survey responses indicate that 81% of firms take into account some form of a target ratio when determining how to raise or retire capital (Graham and Harvey, 2001), the vast majority of whom report a target range rather than a specific target. Thus, our analytic approach is suitable to inform a large number of firms' debt policies.

Overall, our results use general specifications of the ratchet and swap down boundaries, and show that for a wide range of boundaries, we are able to offer a generalized description of the shape of the spread term structure. These results may of course, be specialized to some choice of optimal restructuring boundaries, chosen for example to minimize credit spreads while trading off the expected costs of restructuring. These optimal boundaries may also be chosen, as in past literature, to optimize the value of the firm, trading off debt tax shields versus expected bankruptcy costs, which are not present in our model as they are not required to generate our empirically consistent results.

The rest of the paper proceeds as follows. In Section 2, we present barrier option representations of the various ratchet and swap down combinations that may be embedded in the debt discount. In Section 3, we use our closed-form debt discounts to analyze their sensitivity to parameter values, and we examine the magnitude of and changes to ex-ante spreads when accounting for ratchets and swap downs. We see that these additional features result in specific changes in credit curve level and shapes that match empirical features of spreads better than models without these features. We also discuss optimizing restructuring covenants. In Section 4, we conclude and discuss. For clarity of exposition, we relegate many formulae related to barrier options to the Appendix.

## 2 Model

### 2.1 Overview

We consider seven different cases of firm leverage, the original static debt case in Merton (1974) and six dynamic debt cases, which we describe qualitatively below before presenting the details of our model. Figure 1 provides a visual representation of these cases, which parsimoniously extend the baseline Merton (1974) static debt case.

1. *Original static debt*: We begin with the standard Merton model to price a static debt issue, whereby the debt principal remains fixed throughout the life of the loan. In this case, the discount on a risky debt issue is intuitively captured by the value of a plain vanilla put option on the underlying firm with a strike price equal to the original debt principal. This provides a baseline for comparison with the next six cases of dynamic debt.
2. *Debt with the option to ratchet*: Here, the firm exercises an option to increase its debt if the underlying assets increase sufficiently in value, whereby the firm can use the extra collateral to support more debt, opting for a leveraged buyout of equity shares. In this case, the debt discount is intuitively captured by the combination of: (1) a put option, with a strike price equal to the original debt principal, that is de-activated if the underlying firm value appreciates to an upper barrier (i.e., an up-and-out barrier option), and (2) a put option, with a higher strike price equal to the new increased debt level, that is inactive until the underlying firm value appreciates to an upper barrier (i.e., an up-and-in barrier option).
3. *Debt with the option to swap down*: Analogously, the firm exercises an option to decrease its debt if the underlying assets decrease sufficiently in value, whereby the firm can swap down its debt principal, through an equity issuance. Here, the debt discount is intuitively captured by the combination of: (1) a put option, with a strike price equal to the original debt principal, that is de-activated if the underlying firm value depreciates to a lower barrier (i.e., a down-and-out barrier option), and (2) a put option, with a lower strike price equal to the new decreased debt level, that is inactive until the underlying firm value depreciates to an upper barrier (i.e., a down-and-in barrier option).
4. *Debt with the option to ratchet or swap down*: Here, we combine cases (2) and (3), where the firm exercises in option to either increase or decrease its debt based on whether the underlying assets increase or decrease in value to appropriate trigger levels.
5. *Debt with the option to swap down after ratchet*: In another configuration, we allow the firm an opportunity to reduce its debt after it has exercised its option to increase its debt level. That is, if the underlying firm value increases sufficiently, then the firm can ratchet its debt. Thereafter, if firm value sufficiently declines, then the firm can swap down its (now ratcheted) debt. Intuitively, the debt discount in this case is captured by the combination

of: (1) a put option, with a strike price equal to the original debt principal, that is de-activated if the underlying firm value appreciates to an upper barrier (i.e., an up-and-out barrier option), along with (2) a put option, with a higher strike price equal to the new increased debt level, that is inactive until the underlying firm value appreciates to an upper barrier but is de-activated if the underlying firm value subsequently depreciates sufficiently to some lower barrier (i.e., an up-and-in/down-and-out barrier option), and (3) a put option, with a lower strike price equal to the new decreased debt level, that is inactive until firm value first appreciates to an upper barrier then subsequently decreases to a lower barrier (i.e., an up/down-and-in barrier option).

6. *Debt with the option to ratchet after swap down*: Analogous to case (5), we allow the firm an opportunity to increase its debt after it has exercised its option to decrease its debt level. That is, if the underlying firm value decreases sufficiently, then the firm can swap down its debt. Thereafter, if firm value sufficiently increases, then the firm can ratchet its (now decreased) debt. Intuitively, the debt discount in this case is captured by the combination of: (1) a put option, with a strike price equal to the original debt principal, that is de-activated if the underlying firm value depreciates to a lower barrier (i.e., a down-and-out barrier option), along with (2) a put option, with a lower strike price equal to the new decreased debt level, that is inactive until the underlying firm value decreases to a lower barrier but is de-activated if the underlying firm value subsequently appreciates sufficiently to some upper barrier (i.e., a down-and-in/up-and-out barrier option), and (3) a put option, with a higher strike price equal to the new increased debt level, that is inactive until firm value first depreciates to a lower barrier then subsequently increases to an upper barrier (i.e., a down/up-and-in barrier option).

7. *Debt allowing a swap down after ratchet or vice versa*: Finally, we combine cases (5) and (6), allowing the firm to either ratchet after a swap down, or to swap down after a ratchet.

These parsimonious analytical extensions of the Merton (1974) model are implemented by replacing the standard put option for risky debt discount with combinations of various barrier put options. We now proceed to our formalized framework. Formulae for the various barrier options used are presented in Appendix.

## 2.2 Stochastic Process

To begin, we specify the notation in our model. Let the face value of debt be  $D$ , and we assume it to be zero-coupon with maturity  $T$ . We employ a variant of Merton (1974) as the basis for our model. Discounting takes place at the risk free rate  $r$ , and we posit that the underlying firm value  $V$  follows the usual risk-neutral geometric Brownian motion, i.e.,

$$dV(t) = rV(t) dt + \sigma V(t) dW(t) \quad (1)$$

where the standard deviation is  $\sigma$ , with stochasticity generated by the Weiner process increment  $dW(t) \sim N(0, dt), \forall t$ .

## 2.3 Default Discounts and Spreads

Default is triggered at maturity  $T$  if  $V(T) < D$ , in which case debt holders only recover  $V(T)$ , incurring a loss rate on default of  $[1 - \frac{V(T)}{D}]$ , i.e., one minus the recovery rate on default. The current price of debt at time  $t = 0$  is denoted by function  $B(0)$ . We know from the Merton model that the price of this debt is

$$\begin{aligned} B(0) &= De^{-rT} \cdot N(d_2) + V(0) \cdot N(-d_1) \\ d_1 &= \frac{\ln(V(0)/D) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned} \quad (2)$$

and the credit spread,  $s$ , in this model is

$$\begin{aligned} s &= -\frac{1}{T} \cdot \ln[N(d_2) + N(-d_1)/L(0)] \\ L(t) &= \frac{De^{-r(T-t)}}{V(t)} \end{aligned} \quad (3)$$

Note that  $L(t)$  here is the leverage (i.e., loan-to-value) ratio of the debt in question, accounting for the time value of money. Debt is defined as being underwater when  $L(t) > 1$ . In the Merton model this is possible prior to maturity. We will consider cases where the leverage is initially high,

though not underwater.

The debt discount in the Merton (1974) model is just the price of the plain vanilla put option to default, i.e.,

$$G(0) \equiv P(0) = De^{-rT} \cdot N(-d_2) - V(0) \cdot N(-d_1) \quad (4)$$

And, the price of defaultable debt given above is known to be the price of riskless debt minus this discount, i.e.,  $B(0) = De^{-rT} - G(0)$ , corresponding to equation (2). The credit spread,  $\mathcal{S}$ , in this model is

$$\mathcal{S} = -\frac{1}{T} \ln \left( \frac{B(0)}{De^{-rT}} \right) \quad (5)$$

## 2.4 Deadweight default losses

We may also adjust the Merton model to accommodate deadweight losses on default, i.e., the debt holders get  $\phi V$  on default instead of  $V$ , where  $\phi \leq 1$ . Hence, the default put (or discount value) is based on the following calculation under the risk-neutral measure:

$$P(0) = e^{-rT} \int_0^D [D - \phi V(T)] f(V(T)) dV(T) \quad (6)$$

which simplifies to an adjusted put option formula:

$$G(0) = De^{-rT} \cdot N(-d_2) - \phi V(0) \cdot N(-d_1) \quad (7)$$

This equation is the same as the usual put option formula with no change in the expressions  $N(\cdot)$  for probabilities and only a multiplicative adjustment in just one term, the second half of the Merton formula. As before, the credit spread is given by equation (5).

## 2.5 Modified discounts

We now develop a framework to price debt discounts that depart from the standard Merton model, whereby we account for debt ratchets and swap downs. Formulae for various barrier options we use are provided in the Appendices, and have been modified to accommodate deadweight costs of default, where recovery rates,  $\phi$ , are less than 1.

### 2.5.1 Discount with Debt Ratchet

Assume that when the firm value  $V$  rises to an exogenous level  $D/K$ , we ratchet up debt to a level  $D(1 + \delta)$  and pay down equity with the proceeds, thereby enhancing leverage in the capital structure. Here, we define  $K < 1$  as the  $D/V$  (loan-to-value) level at which the ratchet is triggered. Figure 1 provides a visual depiction.

To keep matters simple, we normalize the value of  $V$  to 1. We may start with an initial leverage ( $D/V$ ) ratio of 0.85 ( $= D$ ), and if  $K = 0.75$ , then at  $V = D/K = 0.85/0.75 = 1.133$ , an appreciation in  $V$  of 13.33 percent, the ratchet occurs, and debt increases by, say, 10%, i.e.,  $\delta = 0.10$ . The price of the new discount, to current debt holders, is now dependent on this potential increase in debt, and may be written as a portfolio of the following barrier options:

$$G_{R,noW}(0) = P_{uo}[V, D; D/K] + \left( \frac{1}{1 + \delta} \cdot P_{ui}[V, D(1 + \delta); D/K] \right) \quad (8)$$

where the first term in the subscript  $\{R, noW\}$  on discount  $G$  stands for whether debt ratchets are allowed and the second term for whether principal swap downs (write downs) are allowed. We use this convention throughout the paper.

In contrast to a plain vanilla put,  $P_{uo}[V, D; D/K]$  stands for an up-and-out put option with strike  $D$  that is knocked out when  $V$  rises to barrier  $D/K$ . Likewise,  $P_{ui}[V, D(1 + \delta); D/K]$  stands for a up-and-in put option with strike  $D(1 + \delta)$  that is knocked in when  $V$  rises to barrier  $D/K$ .

Intuitively, when leverage drops to a level,  $K$ , that merits a ratchet, the original debt discount (characterized by a put option with strike  $D$ ) is cancelled, and a new debt discount (characterized by a put option with strike  $D(1 + \delta)$ ) is written to account for the new debt load. Since  $P_{ui}[V, D(1 + \delta); D/K]$  represents the knocked-in discount on the new, increased debt level, we multiply this by  $\frac{D}{D(1 + \delta)} = \frac{1}{1 + \delta}$  to capture the portion required to guarantee the current liability level,  $D$ .

Thus, using barrier put options instead of vanilla puts, we capture the ex-ante discount value accounting for a possible ratchet. The exact formulae for the up-and-out and up-and-in puts is provided in Appendix A.

### 2.5.2 Discounts with a Debt Swap Down

Using barrier options, we also price debt discounts that account for the latent option to swap down debt principal and dial down leverage as the firm's assets drop in value. That is, assume that when the underlying firm value  $V$  declines to an exogenously determined lower barrier  $D/M$ , we swap down debt, through a debt-equity exchange, reducing the loan principal to  $D(1 - d)$ ,  $0 < d < 1$ . Here,  $M > 1$  is the  $D/V$  level at which the swap down is triggered.

This new discount can be priced at time  $t = 0$  as follows:

$$G_{noR,W}(0) = P_{do}[V, D; D/M] + \left( \frac{1}{1-d} \cdot P_{di}[V, D(1-d); D/M] \right) \quad (9)$$

In contrast to the  $P_{uo}$  and  $P_{ui}$  options introduced in the previous subsection, the  $P_{do}$  and  $P_{di}$  options represent down-and-out and down-and-in puts, whereby  $P_{do}[V, D; D/M]$  represents a barrier put option with strike  $D$  that is knocked out when  $V$  drops to  $D/M$ , and  $P_{di}[V, D(1-d); D/M]$  represents a barrier put with strike  $D(1-d)$  that is knocked in when  $V$  drops to  $D/M$ . Analogous to the previous case (where we allowed for debt ratchets), we multiply this knocked-in discount by a factor of  $\frac{1}{1-d}$  to capture the portion of the discount relevant to the current debt holders'  $D$  level.

The exact formulae for the down-and-out and down-and-in puts with deadweight costs of default are provided in Appendix A.

The automatic swap down of debt when the asset value falls has received a spate of recent attention in the literature on contingent capital based on stock price triggers, see Flannery (2002); Flannery (2009); Pennacchi (2010). In this setting, Sundaresan and Wang (2013) point out that there is no equilibrium because the stock price will jump around as investor's expectations of a conversion change with the stock price. Various approaches suggest remedies to this issue, and there is a series of papers that recommend alternate triggers: Pennacchi, Vermaelen, and Wolff (2010); Glasserman and Nouri (2012); McDonald (2013). Since our trigger for the swap down in this paper is based on asset values, and not the stock price, we avoid running afoul of this multiple equilibria problem, see Appendix B for details.

### 2.5.3 Discount with the Option to Either Ratchet or Swap Down Debt

In this setting, we permit one adjustment to the original debt principal, depending on which case occurs first. If  $V$  rises and touches  $D/K$ , then leverage is increased by ratcheting debt up to level  $D(1 + \delta)$ , after which no further ratchets or swap downs are allowed. On the other hand, if  $V$  falls to level  $D/M$ , then a principal swap down is undertaken, and debt is reduced to  $D(1 - d)$ , after which no further swap downs or ratchets are permitted.

Given that  $D/K > V > D > D(1 + \delta)/M > D/M > D(1 - d)/M$ , the debt discount can be expressed as follows (in this case a fully analytic solution does not exist and we express the result as an integral that embeds a one-touch double barrier binary option):

$$\begin{aligned}
G_{RorW}(0) &= P_{uo/do}[V, D; D/K, D/M] \\
&+ \frac{1}{1-d} \int_0^T f_{D/M, -D/K}(t) \cdot G(t; D/M, D(1-d)) dt \\
&+ \frac{1}{1+\delta} \int_0^T f_{D/K, -D/M}(t) \cdot G(t; D/K, D(1+\delta)) dt
\end{aligned} \tag{10}$$

which we implement as follows:

$$\begin{aligned}
&= P_{uo/do}[V, D; D/K, D/M] \\
&+ \frac{1}{1-d} \sum_{t=dt}^T G(t; D/M, D(1-d)) \cdot [F_{D/M, -D/K}(t) - F_{D/M, -D/K}(t-dt)] \\
&+ \frac{1}{1+\delta} \sum_{t=dt}^T G(t; D/K, D(1+\delta)) \cdot [F_{D/K, -D/M}(t) - F_{D/K, -D/M}(t-dt)]
\end{aligned} \tag{11}$$

This expression has three lines with a double barrier knock-out option in line 1 (which is knocked out upon accessing either barrier, see Appendix D), the value of the restructuring component upon accessing  $D/M$  in line 2, and the value of the ratchet component upon accessing  $D/K$  in line 3. The expressions within are defined as follows:

1.  $f_{H_1, -H_2}(t)$  represents the first-passage probability that  $V_t$  has accessed  $H_1$  for the first time, but has not touched  $H_2$ .  $F_{H_1, -H_2}(t)$  represents the corresponding cumulative density function. In our implementation, we define (approximate)  $dt$  by one-month intervals (i.e.,  $dt = 1/12$ ).

Here we have exploited the fact that this *discounted* first-passage density function is the same as the special case of a one-touch double barrier binary option with a payoff of \$1 (see Appendix F).

2.  $G(t; V, D)$  in line 2 represents the unmodified debt discount from equation (4), which is characterized by a plain vanilla put based on underlying asset value  $V$  and debt level  $D$ . Here, we set  $\{V = D/M; D = D(1 - d)\}$ , since the debt issue is written down to level  $D(1 - d)$  if the firm value drops to  $D/M$ , which leaves us with the following expression:

$$P[D/M, D(1 - d), T - t]$$

3. Again,  $G(t; V, D)$  in line 3 represents the unmodified debt discount from equation (4), which is characterized by a plain vanilla put based on underlying asset value  $V$  and debt level  $D$ . Here, we set  $\{V = D/K; D = D(1 + \delta)\}$ , since the debt issue is ratcheted up to level  $D(1 + \delta)$  if the firm value rises to  $D/K$ , which leaves us with the following expression:

$$P[D/K, D(1 + \delta), T - t]$$

Ultimately, whether  $G_{RorW}(0)$  is greater than or less than the original, unmodified debt discount,  $G(0)$ , depends on the gap between  $K$  and  $M$ , as well the extent to which debt is ratcheted (i.e.,  $\delta$ ) when the underlying firm value accesses the upper barrier versus the extent to which it is reduced (i.e.,  $d$ ) when the firm value accesses the lower barrier.

#### 2.5.4 Discount with the Option to Swap Down Debt After Ratcheting

We now explore how the value of the ratcheted debt discount (presented in section 2.5.1) changes if we account for the option to swap down debt principal after it has been ratcheted. That is, assume that when the firm value  $V$  increases and hits an exogenously determined upper barrier  $D/K$ , we ratchet debt, increasing the loan principal to  $D(1 + \delta)$ . Then, if  $V$  subsequently falls to level  $D(1 + \delta)/M$ , we swap down the ratcheted debt issue, reducing the loan principal to  $D(1 + \delta)(1 - d)$ .

In this instance, the debt discount is priced as follows (the first subscript “*RthenW*” below now

denotes the allowance for a debt ratchet and subsequent swap down):

$$\begin{aligned}
G_{RthenW,noW}(0) &= P_{uo}[V, D; D/K] \\
&+ \frac{1}{1+\delta} P_{ui,do}[V, D(1+\delta); D/K, D(1+\delta)/M] \\
&+ \frac{1}{(1+\delta)(1-d)} P_{udi}[V, D(1+\delta)(1-d); D/K, D(1+\delta)/M]
\end{aligned} \tag{12}$$

where the  $P_{ui,do}$  represents an up-in/down-out put (see Appendix E) that is knocked in when the firm value accesses the upper barrier and is subsequently knocked out if the firm value then depreciates and accesses the lower barrier, and the  $P_{udi}$  represents an up-down-in put that is knocked in only if the underlying firm value accesses the upper barrier then subsequently accesses the lower barrier (priced in Appendix C).

### 2.5.5 Discount with the Option to Ratchet After Swap Down

We also analyze how the value of the restructurable debt discount (presented in section 2.5.2) changes if we account for the option to ratchet debt after it has been written down. In this case, when the firm value  $V$  decreases to an exogenously determined lower barrier  $D/M$ , we swap down debt, decreasing the loan principal to  $D(1-d)$ . Then, if  $V$  subsequently rises to level  $D(1-d)/K$ , we ratchet the reduced debt issue, increasing the loan principal to  $D(1+\delta)(1-d)$ .

In this instance, the debt discount is priced as follows (the second subscript “*WthenR*” below now denotes the allowance for a principal swap down and subsequent ratchet):

$$\begin{aligned}
G_{noR,WthenR}(0) &= P_{do}[V, D; D/M] \\
&+ \frac{1}{1-d} P_{di,uo}[V, D(1-d); D(1-d)/K, D/M] \\
&+ \frac{1}{(1+\delta)(1-d)} P_{dwi}[V, D(1+\delta)(1-d); D(1-d)/K, D/M]
\end{aligned} \tag{13}$$

$P_{di,uo}$  represents a down-in/up-out put (formulated in Appendix E) that is knocked in when the firm value accesses the lower barrier,  $D/M$ , and is subsequently knocked out if the firm value then appreciates and accesses the upper barrier,  $D(1-d)/K$ .  $P_{dwi}$  represents a down-up-in put (formulated in Appendix C) that is knocked in only if the firm value accesses the lower barrier then

subsequently accesses the upper barrier.

### 2.5.6 Discount allowing Ratchet after Swap Down or vice versa

Finally, we price the debt discount accounting for both debt ratchets and swap downs. Specifically, we assume that the debt can either be written down (and then ratcheted thereafter if applicable), or ratcheted (then written down thereafter if applicable).

Formally, if the firm value  $V$  falls to level  $D/M$ , we swap down the principal, reducing the debt to level  $D(1-d)$ . After that if  $V$  rises to level  $D(1-d)/K$ , then we ratchet up debt to  $D(1+\delta)(1-d)$ . On the other hand, if  $V$  rises and hits an upper barrier  $D/K$ , we ratchet up debt to a level  $D(1+\delta)$ . Then, if the firm value subsequently falls to level  $D(1+\delta)/M$ , we swap down the debt principal to level  $D(1+\delta)(1-d)$ .

Given that  $D/K > V > D > D(1+\delta)/M > D/M > D(1-d)/M$ , the debt discount can be expressed as follows (in this case a fully analytic solution does not exist and we express the result as an integral):

$$\begin{aligned}
G_{RthenW,WthenR}(0) &= P_{uo/do}[V, D; D/K, D/M] \\
&+ \frac{1}{1-d} \int_0^T f_{D/M, -D/K}(t) \cdot G_{R,noW}(t; D/M, D(1-d)) dt \\
&+ \frac{1}{1+\delta} \int_0^T f_{D/K, -D/M}(t) \cdot G_{noR,W}(t; D/K, D(1+\delta)) dt \quad (14)
\end{aligned}$$

which we implement as follows:

$$\begin{aligned}
G_{RthenW,WthenR}(0) &= P_{uo/do}[V, D; D/K, D/M] \quad (15) \\
&+ \frac{1}{1-d} \sum_{t=dt}^T G_{R,noW}(t; D/M, D(1-d)) \cdot [F_{D/M, -D/K}(t) - F_{D/M, -D/K}(t-dt)] \\
&+ \frac{1}{1+\delta} \sum_{t=dt}^T G_{noR,W}(t; D/K, D(1+\delta)) \cdot [F_{D/K, -D/M}(t) - F_{D/K, -D/M}(t-dt)]
\end{aligned}$$

This expression has three lines with a double barrier knock-out option in line 1 (which is knocked out by accessing either barrier, and is formulated in Appendix D), the value of the swapdown-then-

ratchet component upon accessing  $D/M$  in line 2, and the value of the ratchet-then-swapdown component upon accessing  $D/K$  in line 3. The expressions within are defined as follows:

1.  $f_{H_1, \neg H_2}(t)$  represents the first-passage probability that  $V_t$  has accessed  $H_1$  for the first time, but has not touched  $H_2$ .  $F_{H_1, \neg H_2}(t)$  represents the corresponding cumulative density function. In our implementation, we define  $dt$  by one-month intervals (i.e.,  $dt = 1/12$ ). This (present-valued) first-passage density function is analogous to holding the special case of a one-touch double barrier binary option with a payoff of \$1 (see Appendix F).
2.  $G_{R, noR}(t; V, D)$  represents the modified debt discount with ratchets from equation (8), which is characterized by a combination of single barrier down-out and down-in put options. Here, we set  $\{V = D/M; D = D(1 - d)\}$ , since the debt issue is written down and decreased to level  $D(1 - d)$  if the firm value decreases to  $D/M$ . This new debt discount is subsequently knocked out if the firm value later increases to level  $D(1 - d)/K$ , whereby the debt issue (which is now at level  $D(1 - d)$ ) is ratcheted to level  $D(1 + \delta)(1 - d)$ , thus knocking in a new discount. This sequence leaves us with the following expression:

$$P_{uo}[D/M, D(1 - d), T - t; D(1 - d)/K] \\ + P_{ui}[D/M, D(1 + \delta)(1 - d), T - t; D(1 - d)/K]$$

3.  $G_{noR, R}(t; V, D)$  represents the modified debt discount with swap downs from equation (9), which is characterized by a combination of single barrier down-out and down-in put options. Here, we set  $\{V = D/K; D = D(1 + \delta)\}$ , since the debt issue is ratcheted and increased to level  $D(1 + \delta)$  if the firm value increases to  $D/K$ . This new debt discount is subsequently knocked out if the firm value later drops to level  $D(1 + \delta)/M$ , whereby the debt principal (which is now at level  $D(1 + \delta)$ ) is written down to level  $D(1 + \delta)(1 - d)$ , thus knocking in a new discount. This sequence leaves us with the following expression:

$$P_{do}[D/K, D(1 + \delta), T - t; D(1 + \delta)/M] \\ + P_{di}[D/K, D(1 + \delta)(1 - d), T - t; D(1 + \delta)/M]$$

We note that using the functions  $F_{H1, \neg H2}(t)$  described above, we may extend the analysis to more complex, ongoing capital structure formulations. For example, firms may wish to commit to a strategy where the stipulation is that if  $V$  reaches the upper boundary  $D/K$  first they will ratchet debt, but then also allow for a subsequent swap down, followed by another ratchet as well. Or if  $V$  reaches the lower boundary  $D/M$  first, the firm will swap down debt, but then also allow for a subsequent ratchet, followed by another swap down, if applicable. The discrete-time formula for the debt discount under this situation is as follows:

$$\begin{aligned}
& G_{RthenW, WthenR}(0) \\
= & P_{uo/do}[V, D; D/K, D/M] \\
+ & \frac{1}{1-d} \sum_{t=dt}^T G_{RthenW, noW}(t; D/M, D(1-d)) \cdot [F_{D/M, \neg D/K}(t) - F_{D/M, \neg D/K}(t-dt)] \\
+ & \frac{1}{1+\delta} \sum_{t=dt}^T G_{noR, WthenR}(t; D/K, D(1+\delta)) \cdot [F_{D/K, \neg D/M}(t) - F_{D/K, \neg D/M}(t-dt)]
\end{aligned}$$

where the formulae for  $G_{RthenW, noW}(t; D/M, D(1-d))$  is given in Section 2.5.4, and the formula for  $G_{noR, WthenR}(t; D/K, D(1+\delta))$  is given in Section 2.5.5.

This summation (or integration approach), through the use of the option formula in Appendix F as a proxy for a discounted first-passage time density, allows recursive computation of discounts with multiple ratchets and swap downs, and supports other nested cases as needed.

### 3 Analysis

The notation and baseline parameters are introduced here for various ensuing numerical analyses. We normalize the underlying firm value to  $V = 1$ , and we compare two debt levels,  $D = \{0.75, 0.50\}$ , representing high-leverage and medium-leverage cases, respectively. We assume an underlying asset volatility of  $\sigma = 20\%$  and a riskless rate of  $r_f = 2\%$ . We begin our analyses assuming a time to maturity of  $T = 15$  years, later considering a range of maturities to map out entire credit curves.

A debt ratchet entails a  $\delta = 30\%$  increase in the current debt principal; analogously, a swap down entails a  $d = 30\%$  decrease. Ratchets occur when  $V$  rises to a level such that  $D/V$  drops

to  $K = 0.40$ ; i.e., ratchets occur at an upper barrier of  $D/K$ . Swap downs occur when  $V$  falls such that  $D/V$  increases to  $M = 1.00$ ; i.e., swap downs occur at a lower barrier of  $D/M$ . In cases where the debt has already been ratcheted, we assume a swap down barrier based on a decrease in leverage relative to the new level of debt. Likewise, in cases where the debt has already been written down, we assume a ratchet barrier based on an increase in leverage relative to the new level of debt.

We consider the seven debt cases described in Section 2.1. A pictorial representation of a sample of these cases is provided in Figure 1.

### 3.1 Debt discounts and credit spreads

In Table 1, we present the debt-discount values under these seven schemes, providing an idea of the relative value of the debt discount structures. Generally speaking, discount values and spreads increase when allowing for debt ratchets, and decrease when allowing for debt swap downs.

With respect to the high-leverage ( $D/V = 0.75$ ) issuer, which we present in Panel A, the original, unmodified Merton model (Case 1) yields a debt discount value of 0.0714, which translates to a credit spread of 92 bps. When we augment this model to allow for a debt ratchet (Case 2), the discount becomes more expensive, with a corresponding spread of 97 bps. On the other hand, when we augment the original model to allow for a principal swap down (Case 3), the discount becomes less expensive, with a corresponding spread of 44 bps. The ratchet effect is tempered when we allow for a follow-on swap down (i.e., (Case 5) < (Case 2)); likewise, the swap down effect is tempered when we allow for a follow-on ratchet (i.e., (Case 6) > (Case 3)).

Similar observations apply to the medium-leverage ( $D/V = 0.50$ ) issuer, which we present in Panel B. Here, the original, unmodified discount (Case 1) is priced at 0.0212, which translates to a credit spread of 39 bps. The credit spread increases to 52 bps when we allow for a debt ratchet (Case 2), and decreases to 16 bps when we allow for a principal swap down (Case 3).

Because ratchets occur at lower leverage, the ratchet effect is more pronounced for medium-leverage issuers than for high-leverage issuers, effecting a 13 bps increase in ex-ante spreads for the medium-leverage issuer (Panel B) in contrast to a 5 bps increase in spreads for the high-leverage issuer (Panel A). Analogously, the swap down effect is more pronounced for high-leverage issuers

than for medium-leverage issuers, effecting a 48 bps decrease in spreads for the high-leverage issuer (Panel A) in contrast to a 23 bps decrease in spreads for the medium-leverage issuer (Panel B).

Overall, debt discount values and ex-ante credit spreads stand to change substantially when considering dynamic versus static debt issues. We now proceed to explore these relations for a range of maturities, assessing the differences not only in the magnitude of spreads but also in the shape of the credit curve.

### 3.2 Credit curves

We plot the term structures of credit spreads for the various combinations of possible debt ratchets and swap downs.

Figure 2 shows credit spreads when the initial leverage ratio is  $D/V = 0.75$ . We observe a classic hump-shaped curve where short-term spreads and long-term spreads are lower than medium-term spreads, under the Merton (1974) model for static debt as well as under the renegotiable debt issue allowing for ratchets. In the upper plot we see, across all maturities, that spreads obtained on debt that can ratchet are always greater than or equal to those obtained from the base case Merton model; conversely, the spreads on debt that can be written down are always lower.

Most notably, the swap down feature increases the slope of the credit curve (in the medium to long end) relative to the static-debt model, consistent with the general upward slope in yield curves that is observed empirically (Helwege and Turner, 1999; Huang and Zhang, 2008). We also observe that the swap down feature brings down spreads noticeably more than the ratchet features increases them, addressing concerns that "newer models tend to severely overstate the credit risk of firms with high leverage" (Eom, Helwege, and Huang, 2004).

The lower panel of Figure 2 compares the base case to more complex combinations where both ratchets and swap downs are allowed, and the other relative comparisons noticed in Table 1 are also borne out in the plots. In all cases, the position of these curves relative to the base case and each other depend on the choice of the leverage barrier parameters  $K$  and  $M$ , as well as the choice of the debt ratchet ( $\delta$ ) or swap down ( $d$ ) proportions at these triggers.

Figure 3 shows credit spreads when the initial leverage ratio is  $D/V = 0.50$ . We observe not

only that spreads are lower from the onset, but also that the credit curves are all upward sloping, whether swap downs are allowed or not. In contrast to the high-leverage issuer, here, we observe that allowing for debt ratchets has an appreciable impact on credit spreads, since a medium-leverage issuer is far more likely to reach a point where the ratchet option is applicable.

We also explore the effect of deadweight costs,  $(1 - \phi)$ , on the level and shape of credit curves. Figures 4 (high leverage) and 5 (medium leverage) demonstrate the difference in credit curves across varying  $\phi$ . As expected, we observe an increase in spreads accompanying decreases in  $\phi$ . Furthermore, the recovery rates affect not only the level of spreads, but also the shape of the curves. Specifically, slopes become steeper in the short end as deadweight costs increase, suggesting that the loss on default incrementally and substantially affects the slope of the credit curve.

Figures 6 (high leverage) and 7 (medium leverage) demonstrate plot spread curves for all seven cases when deadweight costs of default  $(1 - \phi)$  are 30% of firm value. Overall, we observe that even under high deadweight costs, credit curves of high-leverage issuers are still upward sloping when we account for the possibility of a principal swap down, even under high deadweight costs, consistent with the findings of Helwege and Turner (1999) and Huang and Zhang (2008).

### 3.3 Optimal Restructuring Boundaries

Our results thus far present the new spread curves under various dynamics of restructurings entailing debt ratchets and swap downs, setting this paper primarily in the term structure literature (Merton (1974), Black and Cox (1976)). The model we present is distinct from models derived under an optimal capital structure framework (as in Leland (1994)), wherein the objective function is to maximize firm value, which entails a trade-off between debt tax shields and deadweight bankruptcy costs. Instead, the objective function here entails selecting the appropriate restructuring level  $M$ ; i.e., the objective is to designate the leverage (D/V) barrier at which debt will be written down in a way that minimizes credit spreads subject to the rising restructuring costs.

Thus, there is a different kind of tradeoff to consider: a lower  $M$  delivers lower credit spreads but at the same time, incurs greater expected costs of restructuring due to the increased likelihood of triggering the restructuring barrier. We now demonstrate practical implications of our model with regard to how firms, when issuing debt, may optimally select the leverage barrier at which

the debt will be written down in exchange for equity.

To illustrate, Figure 8 shows the varying term structure of spreads under different levels of  $M$ , where we vary  $M$  in the set  $\{0.90, 1.20, 1.50\}$ . The figure's upper panel applies to a high leverage firm with an initial  $D/V = 0.75$ , and the lower panel applies to a medium leverage firm with an initial  $D/V = 0.50$ . We observe that:

1. Spreads increase in  $M$ , as expected, because the sooner restructuring occurs the safer debt is, and
2. Spreads are convex in  $M$ , which we can see by noticing that the difference between the term structures of spreads at  $M = 0.90$  and  $M = 1.20$  is smaller than between spreads at  $M = 1.20$  and  $M = 1.50$ . This is the case for all maturities.

Figure 9 demonstrates how spreads change with the restructuring boundary parameter  $M$  for a fixed time to maturity of  $T = 10$  years (upper plot). As  $M$  increases, we observe that the credit spread increases, since the barrier for restructuring becomes further away from being triggered. Moreover, the marginal change in spreads  $\Delta s$  is always positive, increasing at first and then decreasing. The lower plot demonstrates this effect for initial  $D/V$  ratios of 50% and 75%, respectively. Thus, as was inferred from Figure 8, we now explicitly observe that spreads are not only increasing but also convex in  $M$ , resulting in a marginal spread curve that is hump-shaped.

Therefore, assuming restructuring costs are also convex but decreasing in  $M$  (i.e., costs increase at an increasing rate with the frequency of restructuring), there exists an optimal trade-off between reducing spreads and increasing restructuring costs. As we reduce  $M$  to make debt safer, spreads decline rapidly at first and then at a diminishing rate, whereas restructuring costs increase slowly at first and then more rapidly, suggesting an optimal point at which the marginal cost of restructuring equals the marginal benefit of doing so. Firms can decide the appropriate level of  $M$  as needed using graphs such as those shown in Figures 8 and 9.

For example, take the case where debt swap downs are permitted but debt ratchets are not (i.e., Case 3). As was presented in section 2.5.2, the spread on debt in that case is a function of  $M$ , and

is given by

$$s = -\frac{1}{T} \ln \left[ \frac{De^{-rT} - G(0)}{De^{-rT}} \right]$$

$$G(0) = P_{do}[V, D; D/M] + \frac{1}{1-d} P_{di}[V, D(1-d); D/M]$$

As noted above  $\frac{ds}{dM} \geq 0$ ;  $\frac{d^2s}{dM^2} \geq 0$ . With convex restructuring costs  $c(M)$ , expressed in terms of spread basis points, we have  $\frac{dc}{dM} \leq 0$ , and  $\frac{d^2c}{dM^2} \geq 0$ . The firm may choose to set an optimal restructuring boundary such that a principal swap down occurs when the D/V leverage ratio is equal to  $M^*$ , where  $-ds|_{M=M^*} = dc|_{M=M^*}$ .  $M^*$  may also be chosen so that spreads match the required yields that bond investors require.

To demonstrate, we assume the following restructuring cost function:

$$c(M) = c_0 \exp[-c_1 \cdot M], \quad c_0, c_1 > 0; \quad \frac{dc(M)}{dM} < 0; \quad \frac{d^2c(M)}{dM^2} > 0 \quad (16)$$

Hence,  $c(M)$  is decreasing and convex, as stipulated, and for illustrative purposes, we choose  $c_0 = 50$  and  $c_1 = 1.5$ . Figure 10 plots the cost function against varying  $M$  (top plot).

Using this cost function, Figure 10 also demonstrates the trade-offs entailed when changing  $M$ . The marginal change in expected restructuring cost  $\Delta c$  is always negative, as restructuring becomes less likely as we increase  $M$ . Thus, we see that the optimal point  $M^*$  is reached when  $\Delta s = -\Delta c$ , as shown in the lower plot. For the high leverage firm,  $M^*$  lies around 1.20, whereas for the medium leverage firm,  $M^*$  lies around 1.30. Hence, a more conservative barrier is imposed for the high-leverage firm, where all else equal, spreads are greater, requiring stricter de-leveraging covenants to keep spreads down.

### 3.4 The Credit Spread Puzzle

Much attention has been paid to the credit spread ‘‘puzzle’’. That is: (a) first, the fact that in older models, credit risk accounts for only a small fraction of actual credit spreads (e.g., the Merton (1974), Geske (1977) models), and as pointed out in Huang and Huang (2012), the understatement of spreads occurs mostly for high-grade debt than for low-grade debt. However, in other newer

models, estimated spreads tend to be far too high (the Leland and Toft (1996), Longstaff and Schwartz (1995), and Collin-Dufresne and Goldstein (2001) models), as demonstrated by Eom, Helwege, and Huang (2004).<sup>7</sup> A second part of the puzzle is: (b) the fact that the empirical slope of the term structure of credit spreads is not matched by theoretical models. The slope of the empirical term structure tends to be upward sloping for low quality debt, whereas term structure models tend to generate curves that are downward sloping. Here we examine whether our dynamic extension of the static Merton (1974) model can address some of the puzzle.

In Table 2, we use the same parameters as in Huang and Huang (2012) to replicate their examples,<sup>8</sup> comparing the average empirical yield spread against: 1) model predictions of the base-case Merton model which does not allow debt levels to change, and 2) the model predictions of CDG as well as those of our own when we allow for the possibility that firms may increase or decrease their debt. Like Huang and Huang (2012), we also find that in general, model predictions explain more of the empirically observed spread at longer time horizons and at lower credit ratings. That is, the original Merton model understates spreads, as do the other models with the exception of CDG which consistently overstates spreads. When we allow for possible debt ratchets, our model predictions are able to explain a greater percentage of the spreads observed empirically.<sup>9</sup> However, the ratchets alone do not entirely explain the observed spreads for high quality debt. For instance, model predictions with the ratchet feature are able to explain 42% of Aaa-rate average spreads and 93% of Ba-rated average spreads (compared to the 34% and 76% , respectively, by the base-case Merton model). Hence, while partially resolving some of the credit spread puzzle, dynamic debt models still leave some of the empirical credit spread unexplained.

Overall, our model of dynamic spreads clarifies further both aspects of the credit spread puzzle, i.e., slope and level of the term structure of yield spreads. First, prior models were able to get the empirically evidenced upward slope of the spread curve for investment grade debt correct, but not the upward slope of below investment grade spread curves. Prior models instead obtained humped or downward sloping credit curves for low-grade debt. Our model now generates upward

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<sup>7</sup>See page 512 (Table 3) of Eom, Helwege, and Huang (2004).

<sup>8</sup>See page 170 (Table 2) of Huang and Huang (2012).

<sup>9</sup>As noted by Eom, Helwege, and Huang (2004), the CDG model greatly overstates spreads. Our replication in Table 2 finds the same result. However, this may be remedied by changing the calibration parameters from those in their paper.

sloping curves even for low-grade debt (as shown, for example, in Figure 2 of the paper). Second, the newer models tend to overstate the level of spreads, as shown by Eom, Helwege, and Huang (2004), as well as in our Table 2, where we see that CDG overstates average spreads by up to 381%. Our model has three toggles that enable better fitting of the level of the curve: the levels of deadweight loss, ratchet, and swap down. Unlike CDG, where leverage oscillates evenly around the mean leverage, our leverage barriers for ratcheting up debt and swapping it down may be established asymmetrically changing the probabilities of an increase (ratchet) or decrease (swap down) of debt, allowing for better tuning of the spread term structure. In this vein we find (examples in Table 2) that tuning of the level of spreads is better implemented for high-grade debt through varying the deadweight loss and ratchet parameters, and for low-grade by varying ratchet and swap down parameters.

## 4 Concluding Discussion

We extend the Merton (1974) and Merton (1977) models by developing analytic expressions for the ex-ante pricing of debt discounts where debt principal may be ratcheted up or written down at future dates based on changes in underlying firm value. We develop a novel approach within which these features can be explicitly incorporated into the pricing model using various single- and double-barrier option formulations, embedding discretely punctuated mean-reversion in capital structure and allowing debt to dynamically change. The same framework may be used to determine optimal restructuring covenants by trading off the reduction in spreads against the expected costs inherent in restructuring.

Two highlighted results of the paper are the following: (a) Swap down covenants reduce the level of credit spreads but increase the slope of the credit curve, transforming downward sloping curves into upward sloping ones. (b) Ratchet covenants increase the level of spreads without dramatically changing the slope of the credit curve. Overall, the main finding of the paper is that the proposed model with dynamic debt can generate term structures of credit spreads that are more consistent with stylized facts in the (U.S.) corporate bond market, resolving to some extent aspects of the credit spread puzzle, see Huang and Huang (2012).

The proposed structural model provides a simple and intuitive way to incorporate the notion

of dynamic debt. The fact that the model has closed-form solutions for yield spreads is also an attractive feature of the model. The main implications of the model are also intuitive and make it an interesting extension of the Merton model.

Although our paper connects with the structural class of credit risk models where restructuring occurs when the firm value reaches a barrier, the same approach might also be applied to reduced form factor models (see Jacobs and Li (2008); Wu and Zhang (2008)), allowing the restructuring to occur when spreads reach a given barrier, and defining hazard rates as a function of capital structure or the level of equity (see Das and Sundaram (2007)).

Furthermore, whereas this paper resides in the literature dealing with the term structure of credit spreads, it is distinct from but has linkages to the literature on optimal capital structure, where the objective function is to maximize firm value, trading off deadweight bankruptcy costs versus debt tax shields. In order to focus on the effects of pre-specified debt covenants and effects on ex-ante spreads, we abstract away from tax-shield effects of restructuring (as in Goldstein, Ju, and Leland (2001)), setting call premiums on debt that may be called early in a recapitalization (see Fischer, Heinkel, and Zechner (1989b)), and the endogenous capital structure models of Leland (1994) and Leland and Toft (1996). Instead we introduce a different objective function, where the costs of restructuring are pitted against the reduction in spreads by setting the restructuring boundary optimally. This aligns the paper directly with the high frequency of debt renegotiation that has been documented empirically (as in Roberts and Sufi (2009a); Nini, Smith, and Sufi (2012)), and with the literature on the shapes of the credit spread term structure (see Eom, Helwege, and Huang (2004)).

In sum, under this framework, we are able to extend extant results in the dynamic debt literature, providing closed-form expressions for the term structure of credit spreads across many different prepackaged debt covenants. We are also able to match empirical stylized features: our model's predicted effect of the ratchet and swap down features is consistent with recent evidence that leverage expectations have a material impact on ex-ante spreads (Flannery, Nikolova, and Öztekin, 2012), and overall, we obtain credit spreads and curves that more closely match prior empirical observations, not only in the shape of the curve but also in the magnitude of the spreads.

## Appendices

### A Single Barrier Option Formulae

We provide the pricing equations for single barrier options here. Note that there are 8 different possible barrier options, based on combinations of calls and puts, in or out, up or down cases. The parameter convention we use for these options is taken from Haug (2006). The following equations feed into the barrier option formulae we use in the paper (the variable  $H$  denotes the single barrier in all cases):

$$\begin{aligned}
 A &= \xi\phi_p V e^{(b-r)T} N(\xi x_1) - \xi D e^{-rT} N(\xi x_1 - \xi\sigma\sqrt{T}) \\
 B &= \xi\phi_p V e^{(b-r)T} N(\xi x_2) - \xi D e^{-rT} N(\xi x_2 - \xi\sigma\sqrt{T}) \\
 C &= \xi\phi_p V e^{(b-r)T} (H/V)^{2(\mu+1)} N(\eta y_1) \\
 &\quad - \xi D e^{-rT} (H/V)^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T}) \\
 D &= \xi\phi_p V e^{(b-r)T} (H/V)^{2(\mu+1)} N(\eta y_2) \\
 &\quad - \xi D e^{-rT} (H/V)^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T}) \\
 E &= D e^{-rT} [N(\eta x_2 - \eta\sigma\sqrt{T}) \\
 &\quad - (H/V)^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})] \\
 F &= D [(H/V)^{\mu+\lambda} N(\eta z) \\
 &\quad + (H/V)^{\mu-\lambda} N(\eta z - 2\eta\lambda\sigma\sqrt{T})]
 \end{aligned}$$

where  $\xi, \eta$  are parameters that are set to values  $\{-1, +1\}$  depending on the type of barrier option being considered. Calls that are down-and-in or down-and-out have  $\xi = 1, \eta = 1$ ; calls that are up-and-in or up-and-out have  $\xi = 1, \eta = -1$ ; puts that are down-and-in or down-and-out have  $\xi = -1, \eta = 1$ ; and puts that are up-and-in or up-and-out have  $\xi = -1, \eta = -1$ . The parameter  $\phi_p$  is one minus the deadweight loss in the firm's value on default. Hence, if the firm has no deadweight loss on default, then  $\phi_p = 1$ , else  $\phi_p < 1$ .

The parameter  $b$  is the cost of carry, i.e., the risk free rate plus/minus any other costs/benefits, but in the absence of dividends, we assume that  $b = r$  in all cases. The other parameters are defined

as follows:

$$\begin{aligned}
x_1 &= \frac{\ln(V/D)}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
x_2 &= \frac{\ln(V/H)}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
y_1 &= \frac{\ln(H^2/(VD))}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
y_2 &= \frac{\ln(H/V)}{\sigma\sqrt{T}} + (1 + \mu)\sigma\sqrt{T} \\
z &= \frac{\ln(H/V)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \\
\mu &= \frac{b - \sigma^2/2}{\sigma^2} \\
\lambda &= \sqrt{\mu^2 + \frac{2r}{\sigma^2}}
\end{aligned}$$

We define the single-barrier options we need as functions of the preceding expressions:

1. Up-and-out put: with barrier  $D/K$ . The following equation holds when  $D < H$ , where  $H = D/K$  is the ratchet level for  $V$  to reach.

$$P_{uo}[V, D; H] = A - C + F$$

with  $\xi = -1$  and  $\eta = -1$ .

2. Up-and-in put: with barrier  $D/K$ . The following equation holds when  $D < H$ , where  $H = D/K$  is the ratchet level for  $V$  to reach.

$$P_{ui}[V, D; H] = C + E$$

with  $\xi = -1$  and  $\eta = -1$ .

## B Equilibria with swap down triggers

There is a technical difference between the issues pointed out by Sundaresan and Wang (2013) (SW) and the set up in our paper. As we will show below, the problems of multiple equilibria (or non existence of equilibrium) when contingent capital (CC) is present do not apply to the debt restructuring case in our paper. We explain in the next few paragraphs.

To begin, we summarize the arguments of SW about the equilibrium issues with CC. Assume a firm with assets  $A$ . It has zero-coupon debt of face value  $B$ , maturing at time  $T$ . The capital structure also has contingent capital of face value  $C$ , that converts into  $m(\cdot)$  shares of stock when the assets of the firm  $A$  drop to a level such that the stock value in the firm's  $n$  shares touches a lower boundary  $K$ , i.e.,  $nS \leq K$  where  $S$  is the stock price. At maturity, the CC is not converted if  $nS_T > K$ , i.e.,

$$nS_T = A_T - B - C > K$$

which implies that

$$A_T > B + K + C$$

Likewise, at  $T$  if the CC is converted then  $S_T = (A_T - B)/(n + m)$ . Since for conversion we need that  $nS_T \leq K$ , then

$$nS_T = n(A_T - B)/(m + n) \leq K$$

which implies that

$$A_T \leq B + K + (m/n)K$$

We now have two cases.

1. If  $C < (m/n)K$ , then when  $A_T$  lies in the range  $[B + K + C, B + K + (m/n)K]$ , both criteria, for conversion and non-conversion are satisfied. The solution is unstable on account of multiple equilibria.
2. If  $C > (m/n)K$ , then when  $A_T$  lies in the range  $[B + K + (m/n)K, B + K + C]$ , then neither criteria for non-conversion or conversion are met, and there is no defined equilibrium, or stock price.

This implies that a unique equilibrium exists only when  $C = (m/n)K$ , or in other words when the number of shares  $m$  that convert from CC is  $m = nC/K$ . Hence, the number of conversion shares  $m$  is not a free parameter and is determined by  $\{n, C, K\}$ .

We now show that in the case in our paper, this condition on  $m$  is always satisfied and we do not have problems of multiple equilibria or non-existence of an equilibrium.

To begin, note that we do not have three sources of capital in our capital structure, just two, equity ( $n$  shares at stock price  $S$ ) and zero-coupon debt  $B$ . When the value of the firm drops to some trigger level, denoted  $K$ , we issue fresh equity to retire a pre-specified amount of debt  $D$ . We have two cases.

1. Case 1: At time  $T$ , if  $A_T > K$ , then the value of equity is  $S_T = (A_T - B)/n$ .
2. Case 2: If  $A_T \leq K$ , then we issue equity to retire  $D$  of debt, keeping  $A_T$  the same. Then the value of the stock in the firm will be

$$S_T = \frac{A_T - (B - D)}{n + m}, \quad \text{where } D = mS_T$$

This condition then becomes

$$S_T = \frac{A_T - B + mS_T}{n + m}$$

which simplifies to  $S_T = (A_T - B)/n$ , the same condition as we had in Case 1. Hence, we do not have problems of multiple equilibria or non-existence of equilibrium.

The simple intuition here is that in the CC case, if the function  $m(\cdot)$  for conversion was prefixed and not precisely equal to  $m = nC/K$  then we have confounded equilibria. In our setting, there is no CC, and the conversion number of shares  $m$  is determined at the time of the rebalancing of the capital structure and shares are issued at the existing price so that there is no change in the average price per share even when the number of shares goes from  $n$  to  $(n + m)$ , keeping the asset value of the firm constant. Actually, this is the same condition that SW require for the existence of a unique equilibrium, and in the case of the restructuring debt-equity swap in our paper, it is exactly met, and results in no issues whatsoever.

## C Double Touch Barrier Options

These options are only knocked in (or knocked out) when the underlying touches the lower (upper) barrier, and then touches the upper (lower) barrier. Since there are down-up and up-down, and calls and puts, there are four such cases. We present only the cases we apply in the paper. Here we have two barriers, the upper barrier  $H_U$ , and the lower barrier  $H_L$ .

1. Up-down-in put: We define this double touch option as follows

$$P_{udi}[V, D; H_U, H_L] = \frac{D}{H_U} C_{ui} \left[ V, \frac{H_U^2}{D}; \frac{H_U^2}{H_L}, -r \right] \quad (17)$$

where the  $(-r)$  denotes the fact that the up-and-in call  $C_{ui}$  is being priced off a stochastic process that has reverse drift than the one in equation (1), i.e.,  $dV(t) = -rV(t) dt + \sigma V(t) dW(t)$ .

To see the equivalence of the LHS and RHS of equation (17), note that when  $V$  hits the upper barrier  $H_U$ , it becomes a down-and-in put ( $P_{di}$ ), which by barrier option symmetry (see Gao, Huang, and Subrahmanyam (2000); Haug (2006)), is equal to the RHS of equation (17). When  $V < H_U$ , both RHS and LHS are not triggered and hence have the same value, i.e., zero. But when  $V = H_U$ , both the RHS and LHS become equal to the value of  $P_{di}[V, D; H_L]$ . See Gao, Huang, and Subrahmanyam (2000) [equations (28) and (29) from that paper], for the reasoning to flip the drift of the process. They show that barrier option symmetry results in

$$P_{di}[V, D; H_L] = \frac{D}{V} C_{ui} \left[ V, \frac{V^2}{D}; \frac{V^2}{H_L}, -r \right] \quad (18)$$

Therefore, we may write the double touch options as function of single barrier options. Using barrier option parity we may also write

$$\begin{aligned} P_{udo}[V, D; H_U, H_L] &= P[V, D] \\ &\quad - P_{udi}[V, D; H_U, H_L] \end{aligned}$$

which allows us to price the “out” versions of these double touch options once we have the pricing for the “in” version.

2. Down-up-in put: We define this double touch option as follows

$$P_{dui}[V, D; H_U, H_L] = \frac{D}{H_L} C_{di} \left[ V, \frac{H_L^2}{D}; \frac{H_L^2}{H_U}, -r \right] \quad (19)$$

This identity is analogous to the one presented in equation (17), and the same proof/logic applies, the crux of which is that when  $V = H_L$ , both the RHS and LHS become equal to the value of  $P_{ui}[V, D; H_U]$ . Likewise, the barrier option parity is

$$\begin{aligned} P_{duo}[V, D; H_U, H_L] &= P[V, D] \\ &\quad - P_{dui}[V, D; H_U, H_L] \end{aligned}$$

Since barrier option symmetry allows us to write double barrier options as functions of single barrier options, the formulae in Appendix A for single barrier options that are modified for deadweight costs also apply to the prices in this appendix, and these equivalences are also adapted to the presence of deadweight default costs.

## D Double Barrier Knock-Out Options

These options are knocked out when either the upper or lower barrier is hit.

Up-out / down-out puts: These options have the same payoff as a plain vanilla put given that neither barrier has been accessed prior to maturity. The pricing equation for this option is as follows:

$$P_{uo/do}[V, D; H_U, H_L] = D e^{-rT} \sum_{n=-\infty}^{\infty} A_1(n) - \phi_p V e^{(b-r)T} \sum_{n=-\infty}^{\infty} A_2(n) \quad (20)$$

where

$$\begin{aligned} A_1(n) &= \left( \frac{H_U^n}{H_L^n} \right)^{\mu_1 - 2} \left( \frac{H_L}{V} \right)^{\mu_2} [N(y_1 - \sigma\sqrt{T}) - N(y_2 - \sigma\sqrt{T})] \\ &\quad - \left( \frac{H_L^{n+1}}{H_U^n V} \right)^{\mu_3 - 2} [N(y_3 - \sigma\sqrt{T}) - N(y_4 - \sigma\sqrt{T})] \\ A_2(n) &= \left( \frac{H_U^n}{H_L^n} \right)^{\mu_1} \left( \frac{H_L}{V} \right)^{\mu_2} [N(y_1) - N(y_2)] \end{aligned}$$

$$\begin{aligned}
& - \left( \frac{H_L^{n+1}}{H_U^n V} \right)^{\mu_3} [N(y_3) - N(y_4)] \\
y_1 &= \frac{1}{\sigma\sqrt{T}} \cdot [\ln(VH_U^{2n}/H_L^{2n+1}) + (b + \sigma^2/2)T] \\
y_2 &= \frac{1}{\sigma\sqrt{T}} \cdot [\ln(VH_U^{2n}/(DH_L^{2n})) + (b + \sigma^2/2)T] \\
y_3 &= \frac{1}{\sigma\sqrt{T}} \cdot [\ln(H_L^{2n+2}/(H_L V H_U^{2n})) + (b + \sigma^2/2)T] \\
y_4 &= \frac{1}{\sigma\sqrt{T}} \cdot [\ln(H_L^{2n+2}/(DVH_U^{2n})) + (b + \sigma^2/2)T] \\
\mu_2 &= 0 \\
\mu_1 &= \mu_3 = \frac{2b}{\sigma^2} + 1
\end{aligned}$$

For implementation purposes the infinite sum is taken in a smaller range from  $[-5, +5]$ , see the suggested implementation in Haug (2006). Note that the second term in equation (20) above has been adapted for deadweight costs by the use of a multiplicative factor  $\phi_p$  defined in Appendix A.

## E In-Out Barrier Options

These options are knocked in upon accessing the first barrier, then knocked out upon accessing the next barrier. In-out options can be expressed as a portfolio of the previously priced single barrier and double-touch barrier options. Specifically, an up-in/down-out put can be expressed as:

$$P_{ui,do}[V, D; H_U, H_L] = P_{udo}(V, D; H_U, H_L) - P_{uo}(V, D; H_U) \quad (21)$$

Through parity relations, where  $P_{uo} = P - P_{ui}$  and  $P_{udo} = P - P_{udi}$ , we may write the above expression as:

$$P_{ui,do}[V, D; H_U, H_L] = P_{ui}(V, D; H_U) - P_{udi}(V, D; H_U, H_L) \quad (22)$$

and a down-in/up-out put can be expressed as:

$$P_{di,uo}[V, D; H_U, H_L] = P_{duo}(V, D; H_U, H_L) - P_{do}(V, D; H_L) \quad (23)$$

or again, by parity

$$P_{di,uo}[V, D; H_U, H_L] = P_{di}(V, D; H_L) - P_{dii}(V, D; H_U, H_L) \quad (24)$$

The formulae here are re-expressed as functions of single and double barrier options that have been adapted for deadweight costs of default in Appendix A and Appendix C, so these are already adjusted for these costs as well.

## F One Touch Double Barrier Binary Options

The results here were derived in Hui (1996). Consider an option with two barriers  $H_1$  and  $H_2$ , with  $H_1 < V < H_2$ , such that the option is knocked out if  $V$  touches  $H_2$  but instantly pays \$1 if  $V$  touches  $H_1$ . This valuation formula forms the building block for computing the ratchet and restructure debt discount value. The equation is as follows:

$$\begin{aligned} P[V; H_1, H_2] &= \int_0^T 1 \cdot e^{-rt} \cdot Prob[V_t = H_1 | V_t < H_2, \forall t] dt \\ &= \left(\frac{V}{H_1}\right)^\alpha \left\{ \sum_{j=1}^{\infty} \frac{2}{j\pi} \left[ \frac{\beta - (j\pi/L)^2 \exp\left[-\frac{1}{2}[(j\pi/L)^2 - \beta]\sigma^2 T\right]}{(j\pi/L)^2 - \beta} \right] \right. \\ &\quad \left. \times \sin\left(\frac{j\pi}{L} \ln \frac{V}{H_1}\right) + \left(1 - \frac{\ln \frac{V}{H_1}}{L}\right) \right\} \end{aligned}$$

where

$$\begin{aligned} L &= \ln\left(\frac{H_2}{H_1}\right) \\ \alpha &= -\frac{1}{2}(k_1 - 1) \\ \beta &= -\frac{1}{4}(k_1 - 1)^2 - \frac{2r}{\sigma^2} \\ k_1 &= \frac{2r}{\sigma^2} \end{aligned}$$

Because this is a digital/binary option and the pay off on this option is \$1, the value here is the expected discount probability that the firm value  $V$  touches the lower barrier  $H_1$  and does not touch  $H_2$ . We will use this formula to derive components of the ratchet and restructure discount.

Note that if we want the converse option, i.e., an option with two barriers  $H_1$  and  $H_2$ , with  $H_1 > V > H_2$ , such that the option is knocked out if  $V$  touches  $H_2$  but instantly pays \$1 if  $V$  touches  $H_1$ , then use the same equation with  $H_1$  and  $H_2$  flipped.

Since the formula in this appendix is only used for computing first passage time density functions and does not involve payoffs, no adjustment needs to be made for deadweight costs of default.

## G Sub-Homogeneity Property of Barrier Options

We state without proof an interesting property of barrier options that may be used to derive bounds in comparing some of the discounts in this paper to others. Assume a barrier option priced using a generic function  $B[V, K; H]$ , where  $H$  is the barrier,  $V$  is the underlying, and  $K$  is the strike. Irrespective of the nature of the option, i.e., put/call, or up/down, or in/out, it is the case that for  $\gamma \in (0, 1)$ , we have the following two inequalities:

$$\begin{aligned} (1 + \gamma)B[V, D; H] &\geq B[V(1 + \gamma), K(1 + \gamma); H(1 + \gamma)] \\ (1 - \gamma)B[V, D; H] &\leq B[V(1 - \gamma), K(1 - \gamma); H(1 - \gamma)] \end{aligned}$$

What this means is that the barrier option is less sensitive than one-for-one. Therefore, if we increase each of  $V, K$ , and  $H$  by 10%, then the option value will increase by less than 10%. Likewise, if we reduce all three inputs by 10%, the option price will drop by less than 10%. We call this property the “sub-homogeneity” of barrier options. This is in contrast to vanilla (non-barrier) options that are homogeneous of degree one in the underlying and the strike.

## References

- Black, Fischer., and John Cox (1976). “Valuing Corporate Securities: Some Effects of Bond Indenture Provisions,” *Journal of Finance* 31(2), 351–367.
- Black, Fischer., and Myron Scholes (1973). “The Pricing of Options and Corporate Liabilities,” *Journal of Political Economy* 81(3), 637–654.
- Collin-Dufresne, Pierre., and Robert Goldstein (2001). “Do Credit Spreads Reflect Stationary Leverage Ratios?” *Journal of Finance* 56(5), 1929–1957.
- Eom, Young Ho., Jean Helwege, Jing-Zhi Huang (2004). “Structural Models of Corporate Bond Pricing: An Empirical Analysis,” *Review of Financial Studies* 17(2), 499–544.
- Das, Sanjiv., and Rangarajan Sundaram (2007). “An Integrated Model for Hybrid Securities,” *Management Science* 53(9), 1439–1451.
- Fischer, Edwin., Robert Heinkel, and Josef Zechner (1989). “Dynamic Capital Structure Choice: Theory and Tests,” *Journal of Finance* 44(1), 19–40.
- Fischer, Edwin., Robert Heinkel, and Josef Zechner (1989). “Dynamic Recapitalization Policies and the Role of Call Premia and Issue Discounts,” *Journal of Financial and Quantitative Analysis* 24(4), 427–446.
- Flannery, Mark (2002), “‘No Pain, No Gain’ Effecting Market Discipline via ‘Reverse Convertible Debentures’”, Working paper, University of Florida, sequently published in Hal S. Scott (ed.), *Capital Adequacy beyond Basel: Banking, Securities, and Insurance*, Oxford University Press, 2005.
- Flannery, Mark (2009). “Stabilizing Large Financial Institutions with Contingent Capital Certificates,” Working paper, University of Florida.
- Flanery, Mark., Stanislava Nikolova, and Özde Öztekin (2012). “Leverage Expectations and Bond Credit Spreads,” *Journal of Financial and Quantitative Analysis* 47(4), 689–714.

- Gao, Bin., Jing-Zhi Huang, and Marti Subrahmanyam (2000). “The Valuation of American Barrier Options using the Decomposition Technique,” *Journal of Economic Dynamics and Control* 24, 1783–1827.
- Garleanu, Nicolae., and Jeff Zwiebel (2009). “Design and Renegotiation of Debt Covenants,” *Review of Financial Studies* 22(2), 749–781.
- Geske, R. (1977). “The Valuation of Corporate Liabilities as Compound Options,” *Journal of Financial and Quantitative Analysis* 12(4), 541–52.
- Glasserman, Paul., and B. Nouri (2012). “Contingent Capital with a Capital-ratio Trigger,” *Management Science* 58, 1816–1833.
- Goldstein, Robert., Nengjui Ju, and Hayne Leland (2001). “An EBIT-Based Model of Dynamic Capital Structure,” *Journal of Business* 74(4), 483–512.
- Graham, John., and Campbell Harvey (2001). “Graham, J.R., Harvey, C.R., 2001. The Theory and Practice of Corporate Finance: Evidence from the Field,” *Journal of Financial Economics* 61, 187–243.
- Han, Bing., and Yi Zhou (2010). “Understanding the Term Structure of Credit Default Swap Spreads,” Working paper, University of Texas, Austin.
- Haug, Espen (2006). “The Complete Guide to Option Pricing Formulas,” Second Edition, McGraw-Hill, New York.
- Helwege, Jean., and Christopher Turner (1999). “The Slope of the Credit Curve for Speculative-Grade Issuers,” *Journal of Finance* 54(5), 1869–1884.
- Huang, Jing-Zhi., and Xiongfei Zhang (2008). “The Slope of Credit Spread Curves,” *Journal of Fixed Income* 18(1), 56–71.
- Huang, Jing-Zhi., and Ming Huang (2012). “How much of the corporate-treasury yield spread is due to credit risk?” *Review of Asset Pricing Studies* 2, 153–202.
- Hui, Cho (1996). “One-touch Double Barrier Binary Option Values,” *Applied Financial Economics* 6, 343–346.

- Jacobs, Kris., and Xiaofei Li (2008). “Modeling the Dynamics of Credit Spreads with Stochastic Volatility,” *Management Science* 54(6), 1176–1188.
- Khandani, Amir E., Lo, Andrew W. and Merton, Robert C., (2009). “Systemic Risk and the Refinancing Ratchet Effect,” MIT Sloan Research Paper No. 4750-09; Harvard Business School Finance Working Paper No. 1472892. Available at SSRN: <http://ssrn.com/abstract=1472892> or <http://dx.doi.org/10.2139/ssrn.1472892>
- Lando, David., and Allan Mortensen (2005). “Revisiting the Slope of the Credit Spread Curve,” *Journal of Investment Management* 3(4), 6–32.
- Leland, Hayne., (1994). “Corporate Debt Value, Bond Covenants, and Optimal Capital Structure,” *Journal of Finance* 49(4), 1213–1252.
- Leland, Hayne E., and Klaus B. Toft (1996). “Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads,” *Journal of Finance* 51(3), 987–1019.
- Longstaff, Francis A., and Eduardo S. Schwartz (1995). “A Simple Approach to Valuing Risky Fixed and Floating Rate Debt,” *Journal of Finance* 50(3), 789–819.
- Madan, Dilip., and Haluk Unal (2000). “A Two Factor Hazard Rate Model for Pricing Risky Debt and the Term Structure of Credit Spreads,” *Journal of Financial and Quantitative Analysis* 35,43–65.
- McDonald, Robert. (2013). “Contingent Capital with a Dual Price Trigger,” *Journal of Financial Stability* 9(2), 230–241.
- Merton, R.C. (1974). “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance* 29, 449–470.
- Merton, R.C. (1977). “An Analytic Derivation of the Cost of Loan Guarantees and Deposit Insurance: An Application of Modern Option Pricing Theory,” *Journal of Banking and Finance* 1, 3–11.
- Nini, Greg., David Smith, and Amir Sufi (2012). “Creditor Control Rights, Corporate Governance, and Firm Value,” *Review of Financial Studies* 25(6), 1713–1761.

- Pennacchi, George (2010). “A Structural Model of Contingent Capital,” Working paper, University of Illinois, Urbana-Champaign.
- Pennacchi, George., Theo Vermaelen, and Christian Wolff (2010). “Contingent Capital: The Case for COERCs,” Working paper, University of Illinois, Urbana-Champaign.
- Piskorski, Tomasz., Amit Seru, and Vikrant Vig (2010). “Securitization and Distressed Loan Renegotiation: Evidence from the Subprime Mortgage Crisis,” *Journal of Financial Economics* 97, 369–397.
- Roberts, Michael R. and Amir Sufi (2009). “Renegotiation of Financial Contracts: Evidence from Private Credit Agreements,” *Journal of Financial Economics* 93, 159–184.
- Roberts, Michael R. and Amir Sufi (2009). “Financial Contracting: A Survey of Empirical Research and Future Directions,” *Annual Reviews in Financial Economics* 1, 1–20.
- Strebulaev, Ilya (2007). “Do Tests of Capital Structure Theory Mean What They Say?” *Journal of Finance* 62(4), 1747–1787.
- Sundaresan, Suresh., and Zhenyu Wang (2013). “On the Design of Contingent Capital with a Market Trigger,” forthcoming *Journal of Finance*.
- Wu, Liuren., and Frank Zhang (2008). “A No-Arbitrage Analysis of Macroeconomic Determinants of the Credit Spread Term Structure,” *Management Science* 54(6), 1160–1175.

Table 1: Credit spreads and discount pricing ( $G$ ) for a debt issue that has a loan principal of  $D = \{0.75, 0.50\}$  and where the firm value is normalized to  $V = 1$ . The remaining loan parameters are:  $T = 15$  years,  $\sigma = 0.20$ , and  $r_f = 0.02$ . Debt ratchets entail a  $\delta = 30\%$  increase in debt level when the firm appreciates in value such that  $D/V$  reaches  $K = 0.40$ , and swap downs entail a  $d = 30\%$  reduction in debt level when the firm depreciates in value such that the  $D/V$  reaches  $M = 1.00$ . Spreads are expressed in basis points. The last column in the table shows the guarantee prices when there is also a deadweight loss on default of 30%, i.e.,  $\phi = 0.7$ .

	$G$	% change	Spread	change	$G_{\phi=0.7}$
Panel A. $D/V = 0.75$					
(1) original	0.0714	—%	92	—	0.1092
(2) ratch, no sdown	0.0753	5.38%	97	5	0.1162
(3) no ratch, sdown	0.0354	-50.47%	44	-48	0.0586
(4) ratch <i>or</i> sdown	0.0404	-43.50%	50	-41	0.0677
(5) ratch then sdown	0.0728	1.90%	94	2	0.1116
(6) sdown then ratch	0.0381	-46.68%	47	-44	0.0638
(7) ratch then wdown, or vice versa	0.0389	-45.54%	48	-43	0.0654
Panel B. $D/V = 0.50$					
(1) original	0.0212	—%	39	—	0.0354
(2) ratch, no sdown	0.0280	32.45%	52	13	0.0467
(3) no ratch, sdown	0.0088	-58.58%	16	-23	0.0159
(4) ratch <i>or</i> sdown	0.0200	-5.39%	37	-2	0.0349
(5) ratch then sdown	0.0212	0.33%	39	0	0.0354
(6) sdown then ratch	0.0095	-55.32%	17	-22	0.0173
(7) ratch then sdown, or vice versa	0.0105	-50.42%	19	-20	0.0190

Table 2: Credit spreads and guarantee pricing for a debt issue under varying parameters. Initial  $D/V$ , rating category,  $\sigma$ , and corresponding empirical average yield spread parameters are from Table 2 (page 170) of Huang and Huang (2012). Firm value is normalized to  $V = 1$ ,  $r_f = 0.02$ , and we assume a fire-sale discount of 10%, i.e.,  $\phi = 0.90$ . For our ratchet/swapdown-based model predictions, debt ratchets (swapdowns) entail a  $\delta = 10\%$  increase (decrease) in debt level when the firm appreciates (or depreciates) in value such that  $D/V$  reaches  $K = \{D/V \pm 0.10\}$ . For the CDG-based model predictions, we set parameters  $\delta = 0.03$ ,  $\lambda = 0.18$ ,  $\nu = 0.60$ , and  $\omega = 0.56$ , as per Collin-Dufresne and Goldstein (2001). Spreads are expressed in basis points. Percentage of empirical spread explained is expressed in parentheses to the right of each model-based spread prediction. (Note: To comport with the parameters used by Huang and Huang (2012), the strike in our model is set to reported  $D/V \cdot e^{rT}$ )

Rating	$D/V$	$\sigma$	$avg_{spread}$	<i>Merton</i>	<i>Merton</i> $_{\phi=0.9}$	<i>CDG</i>	<i>Ratchet</i>	<i>Swapdown</i>	<i>Ratch-or-Sdown</i>
Panel A. $T = 10$ -year horizon									
Aaa	0.131	0.321	63	22 (34%)	26 (42%)	240 (381%)	26 (42%)	21 (33%)	21 (33%)
Aa	0.212	0.284	91	32 (35%)	39 (43%)	237 (260%)	39 (43%)	31 (34%)	32 (35%)
A	0.320	0.256	123	50 (41%)	62 (50%)	263 (214%)	65 (53%)	48 (39%)	54 (44%)
Baa	0.433	0.258	194	98 (51%)	118 (61%)	355 (183%)	128 (66%)	96 (49%)	109 (56%)
Ba	0.535	0.324	320	244 (76%)	274 (86%)	581 (182%)	297 (93%)	240 (75%)	266 (83%)
B	0.657	0.395	470	443 (94%)	484 (103%)	734 (156%)	518 (110%)	440 (94%)	476 (101%)
Panel B. $T = 4$ -year horizon									
Aaa	0.131	0.362	55	3 (6%)	3 (6%)	66 (120%)	3 (6%)	2 (3%)	2 (3%)
Aa	0.212	0.344	65	14 (21%)	19 (29%)	140 (215%)	19 (29%)	13 (19%)	13 (19%)
A	0.320	0.298	96	24 (25%)	34 (36%)	191 (199%)	35 (36%)	23 (24%)	24 (25%)
Baa	0.433	0.289	158	63 (40%)	85 (54%)	350 (222%)	91 (57%)	60 (38%)	67 (43%)
Ba	0.535	0.343	320	206 (64%)	254 (79%)	796 (249%)	278 (87%)	200 (63%)	232 (72%)
B	0.657	0.396	470	428 (91%)	500 (106%)	1,288 (274%)	550 (117%)	421 (90%)	479 (102%)

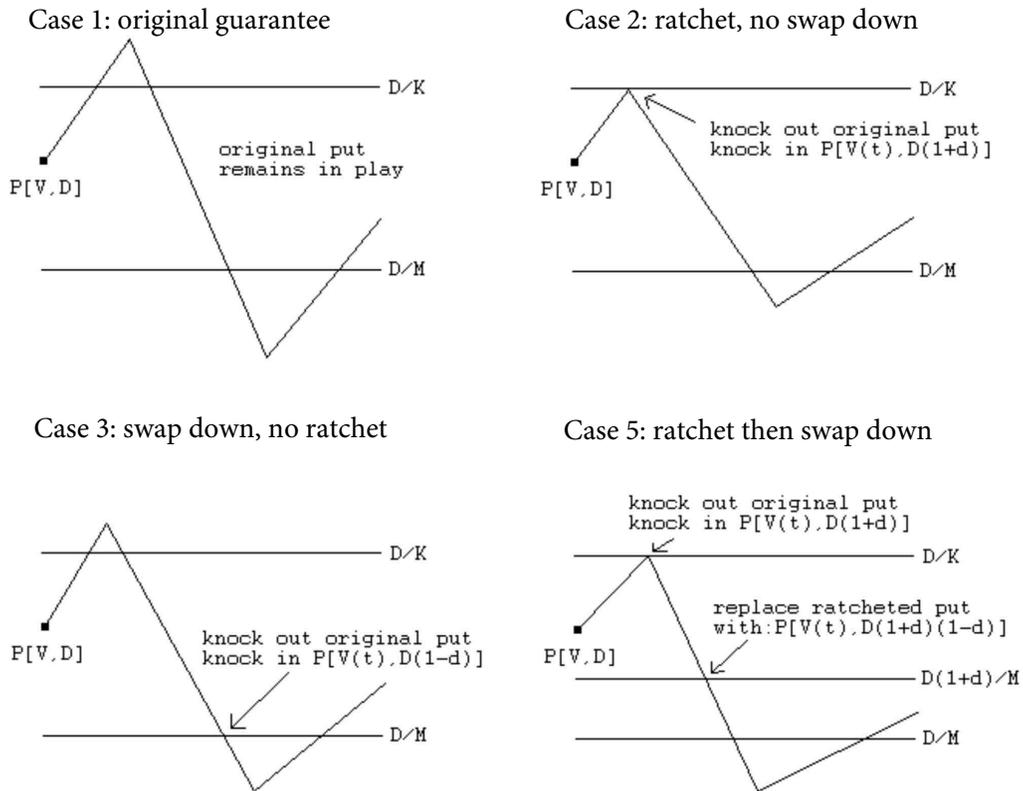


Figure 1: This figure provides a pictorial representation of what happens to the various debt issues we consider as the underlying asset appreciates or depreciates in value.

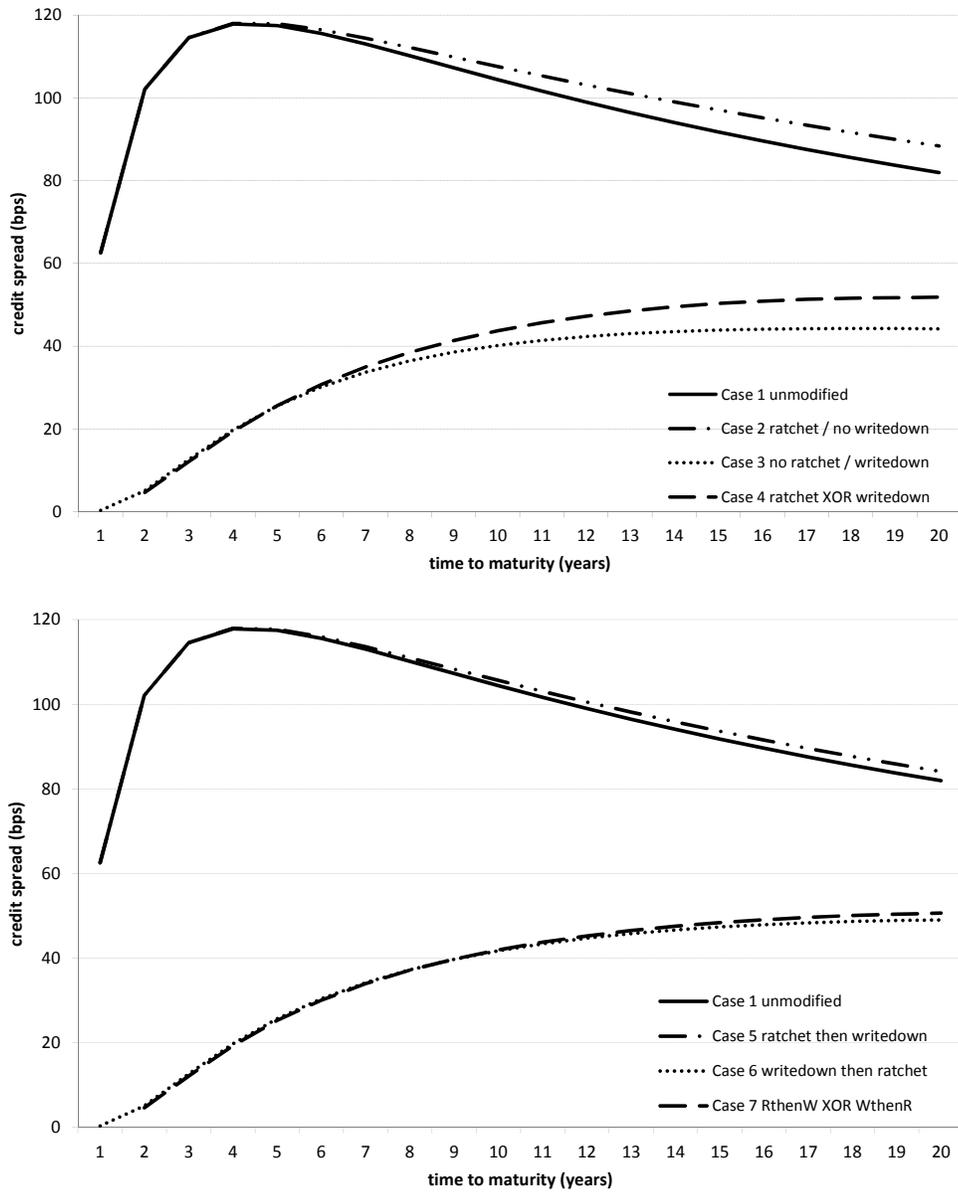


Figure 2: This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of  $D/V = 0.75$ , with a target band of  $K = 0.40$  and  $M = 1.00$ . Ratchets entail a 30% increase in debt level and swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.

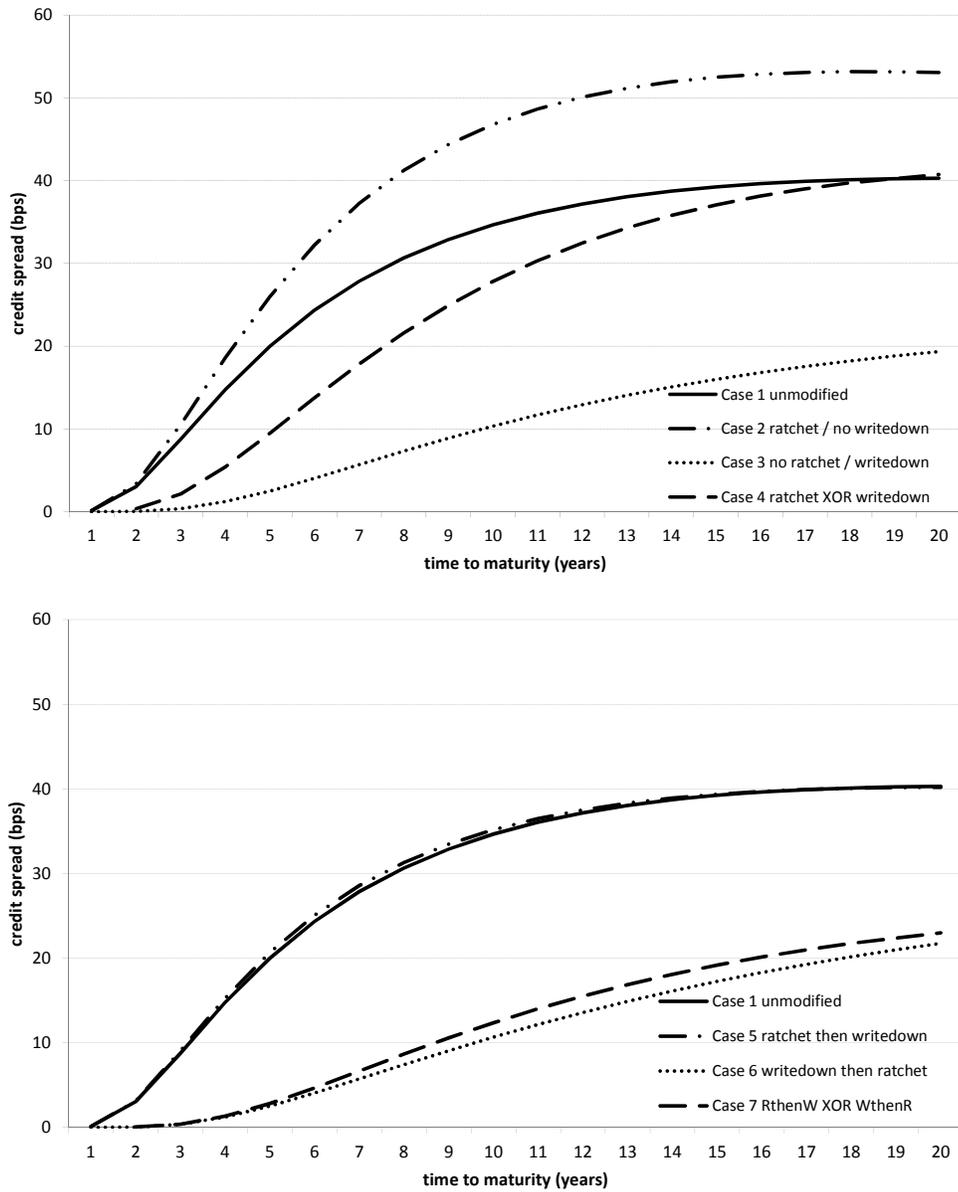


Figure 3: This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of  $D/V = 0.50$ , with a target band of  $K = 0.40$  and  $M = 1.00$ . Ratchets entail a 30% increase in debt level and swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.

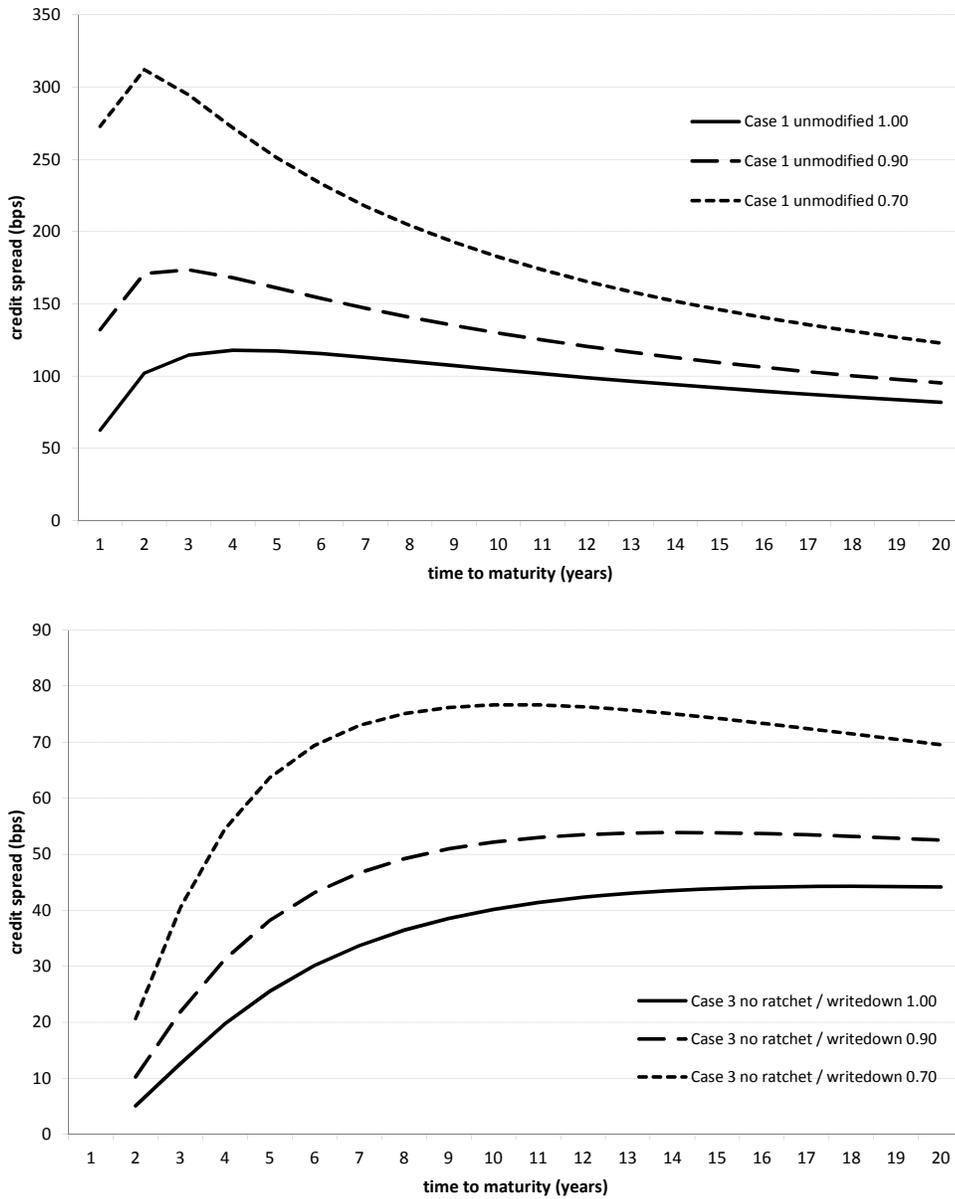


Figure 4: Comparing credit spreads across various deadweight costs. This figure plots the term structure of credit spreads under a current leverage ratio of  $D/V = 0.75$ , with a target band of  $K = 0.40$  and  $M = 1.00$ . Ratchets entail a 30% increase in debt level and swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.

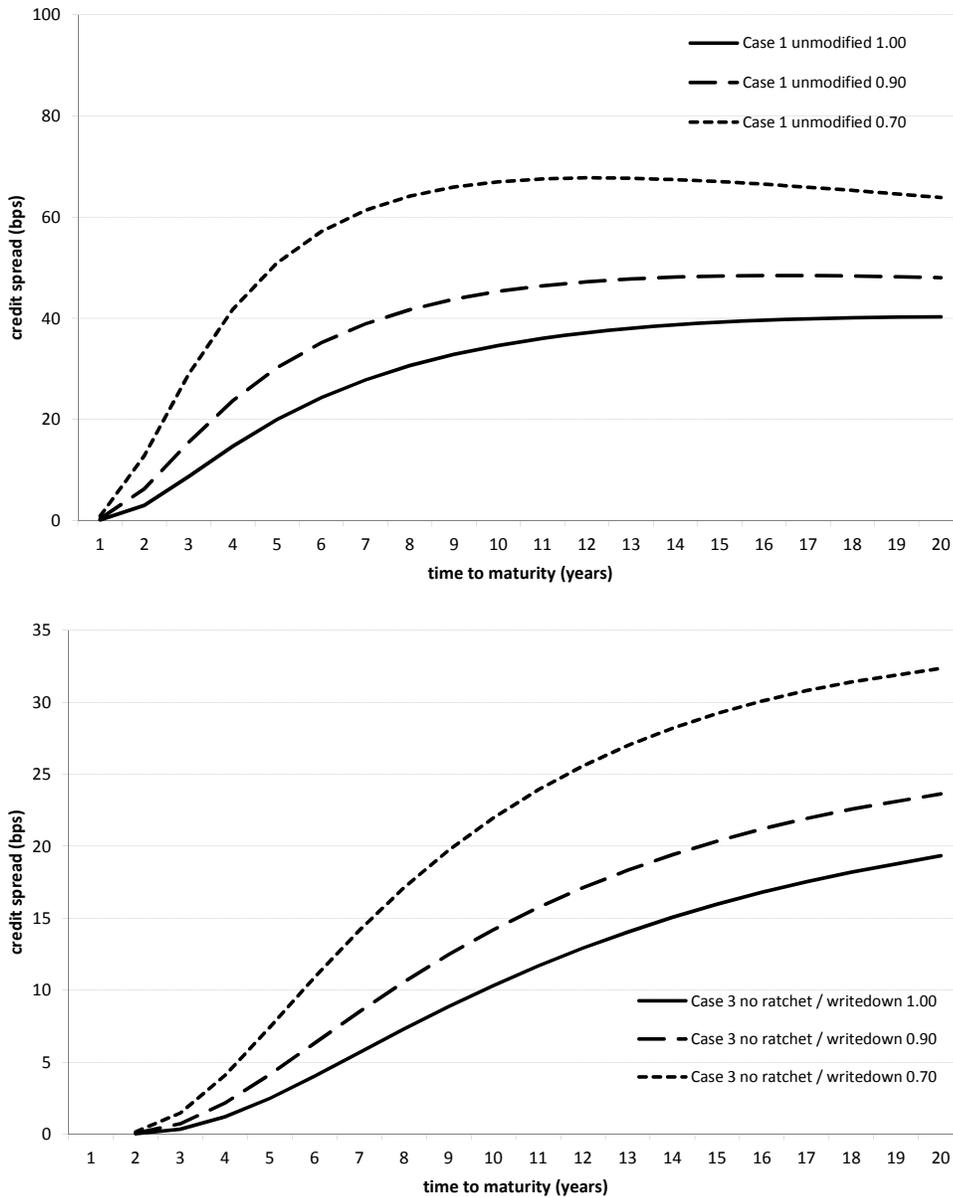


Figure 5: Comparing credit spreads across various deadweight costs. This figure plots the term structure of credit spreads under a current leverage ratio of  $D/V = 0.50$ , with a target band of  $K = 0.40$  and  $M = 1.00$ . Ratchets entail a 30% increase in debt level and swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.

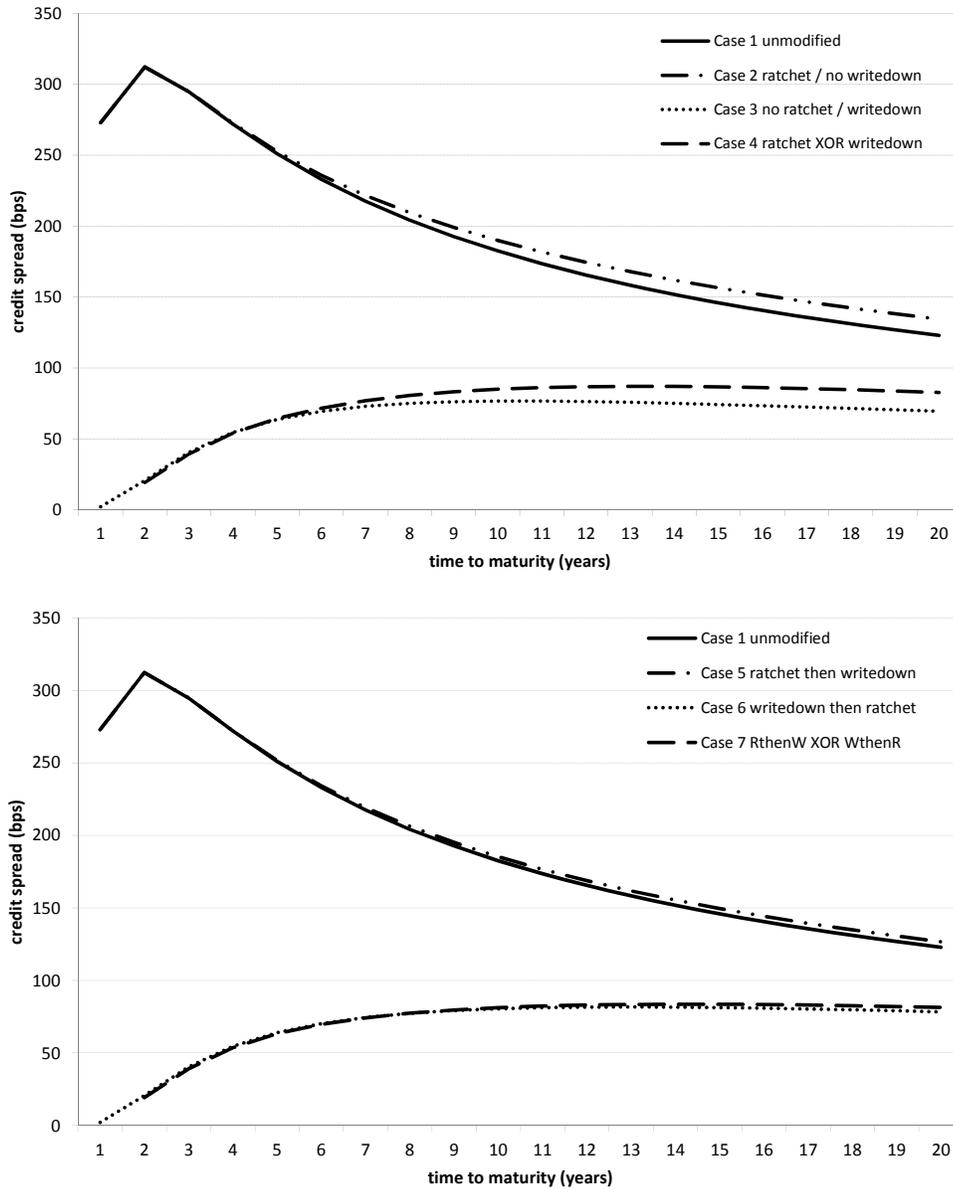


Figure 6: Credit spreads with 30% deadweight costs on default, i.e.,  $\phi = 0.70$ . This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of  $D/V = 0.75$ , with a target band of  $K = 0.40$  and  $M = 1.00$ . Ratchets entail a 30% increase in debt level and swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.

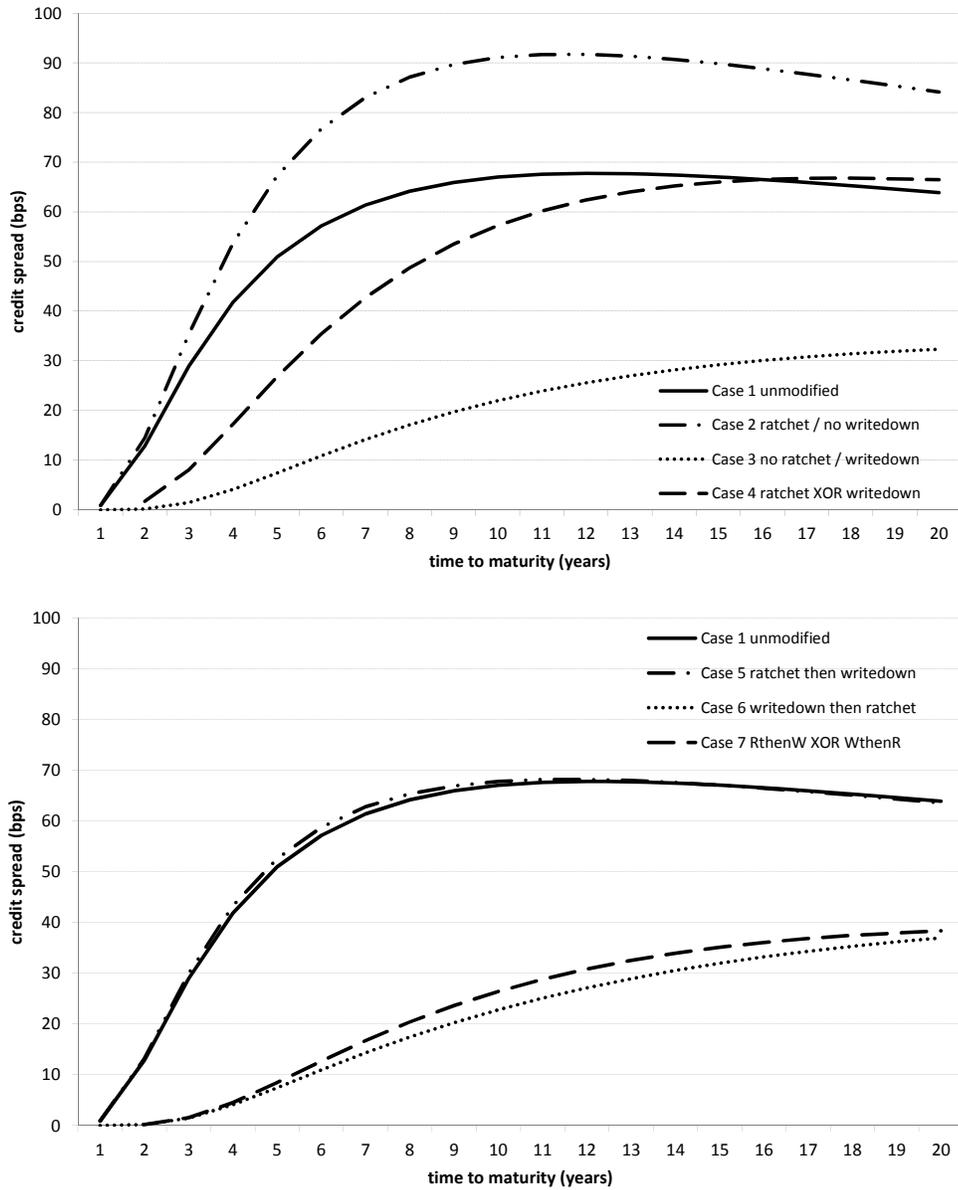


Figure 7: Credit spreads with 30% deadweight costs on default, i.e.,  $\phi = 0.70$ . This figure plots the term structure of credit spreads for each of our various cases under a current leverage ratio of  $D/V = 0.50$ , with a target band of  $K = 0.40$  and  $M = 1.00$ . Ratchets entail a 30% increase in debt level and swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20% and a risk free rate of 2%.

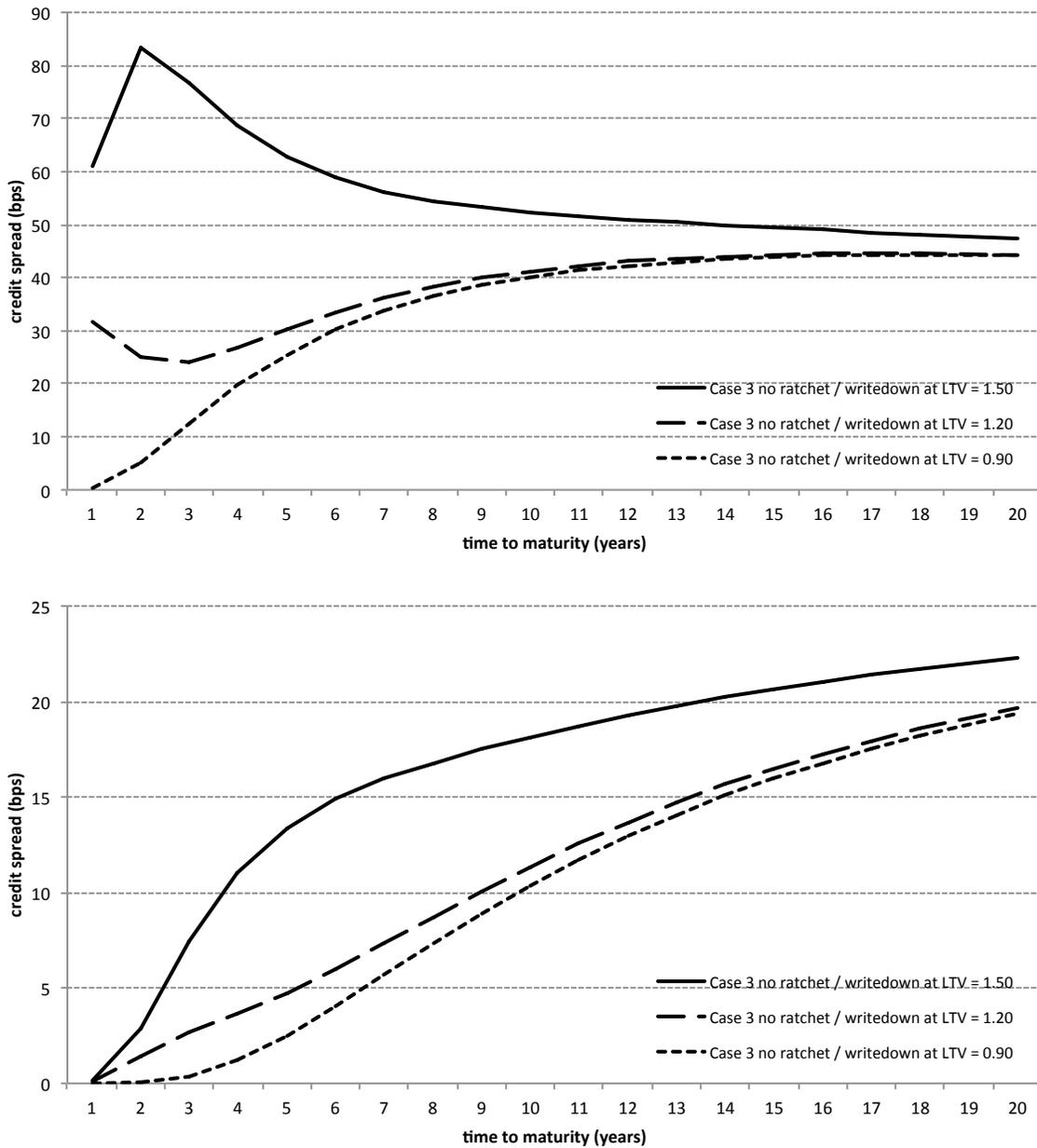


Figure 8: Comparing spread curves across various swap down barriers. This figure plots the term structure of credit spreads under current leverage ratios of  $D/V = 0.75$  (top figure) and  $D/V = 0.50$  (bottom figure), whereby a swap down is triggered at  $M = \{1.50, 1.20, 0.90\}$ . swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20%, a risk free rate of 2%, and deadweight costs of  $\phi = 0.00$ .

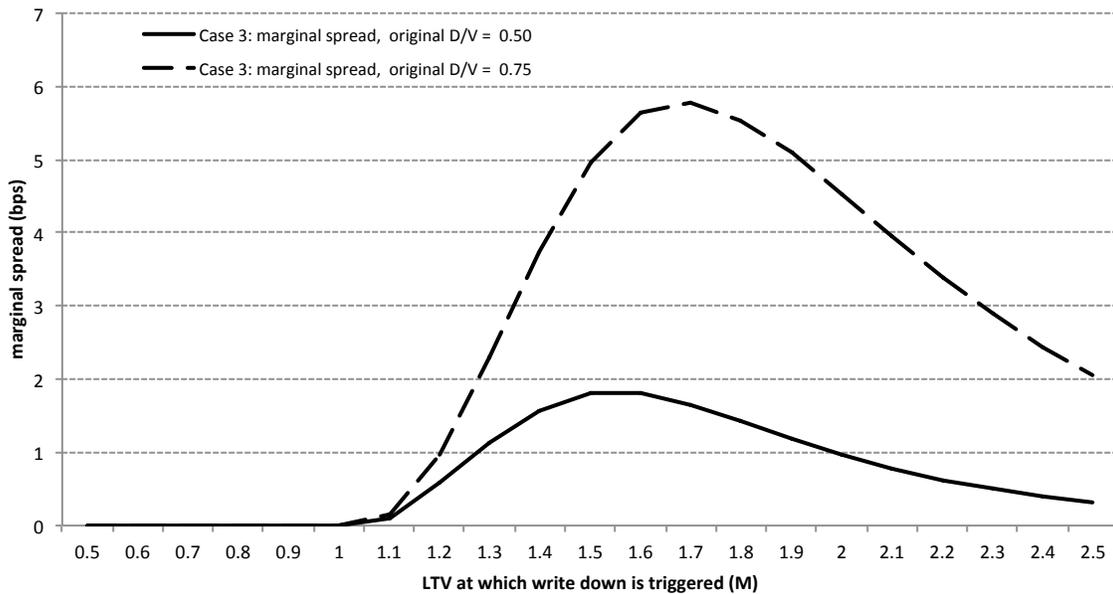
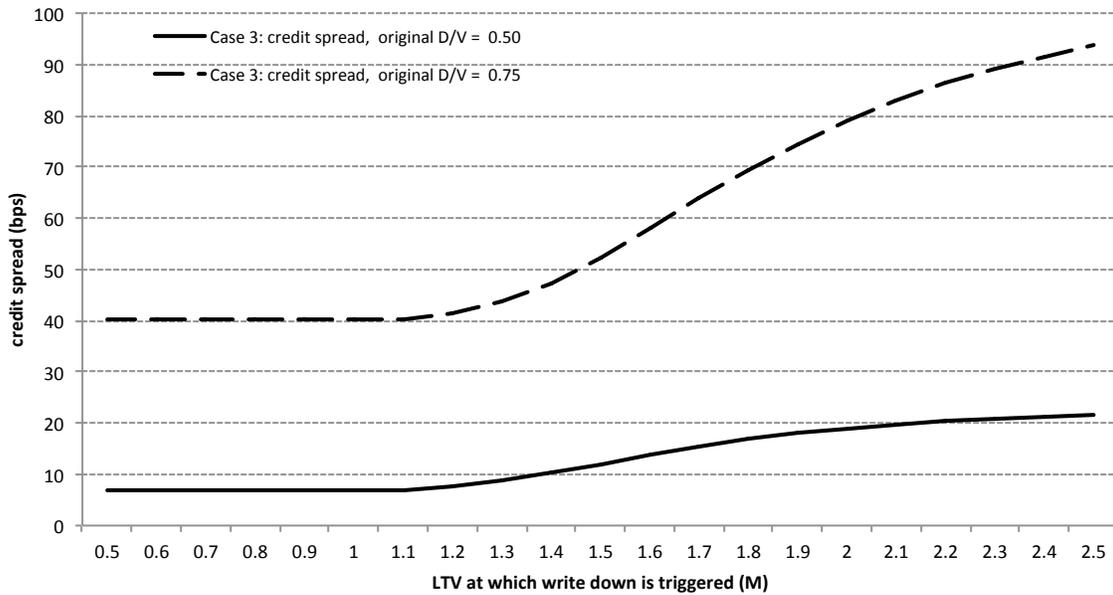


Figure 9: Credit spreads and marginal spreads plotted against restructuring barrier  $M$  (i.e., the D/V leverage ratio at which a swap down is triggered). We assume an underlying asset volatility of 20%, a risk free rate of 2%, time to maturity of  $T = 10$  years, and deadweight costs of  $\phi = 0.00$ .

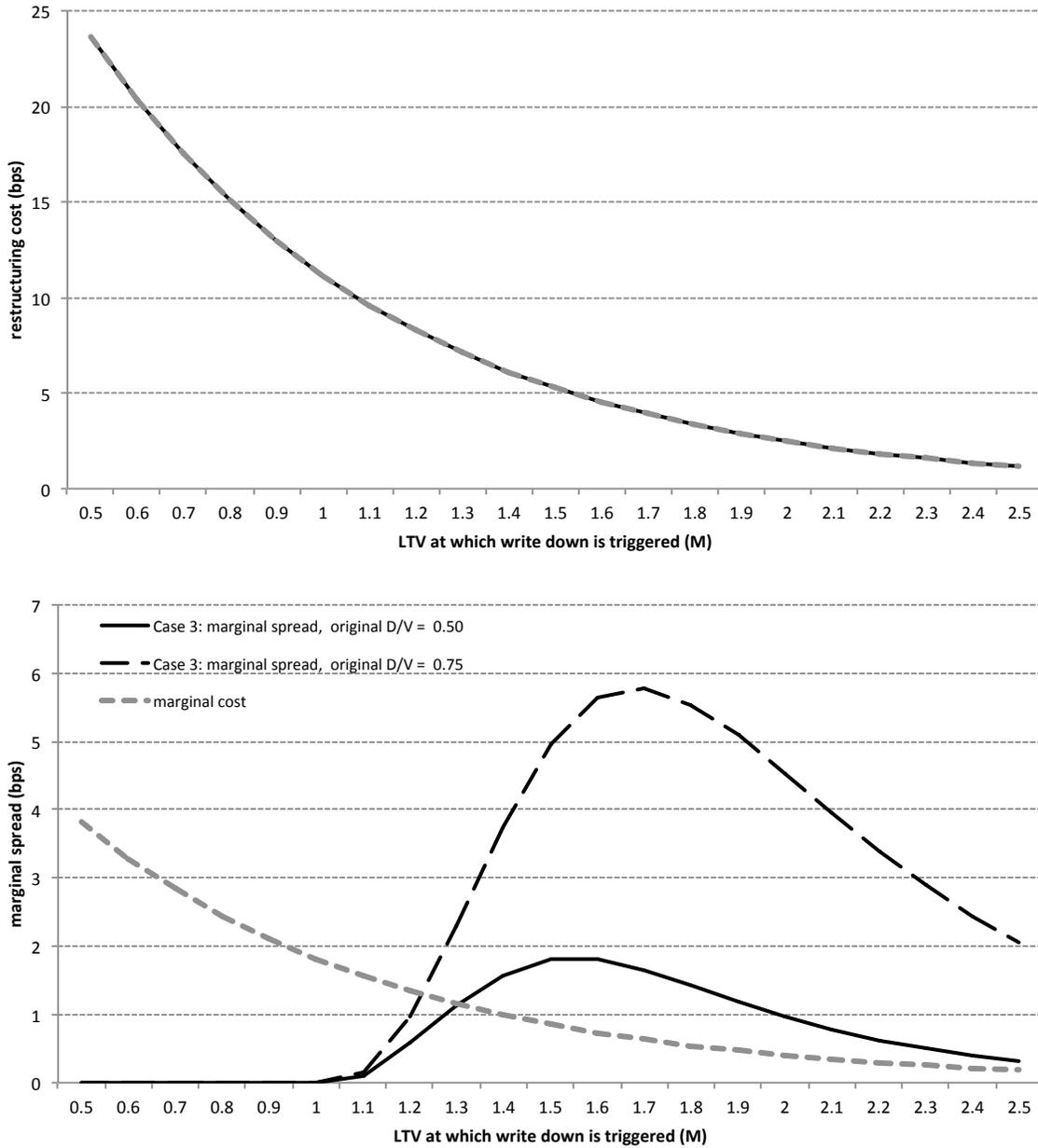


Figure 10: Cost function, marginal credit spreads, and marginal costs plotted against restructuring barrier  $M$  (i.e., the D/V leverage ratio at which a swap down is triggered). Swap downs entail a 30% reduction in debt level. We assume an underlying asset volatility of 20%, a risk free rate of 2%, time to maturity of  $T = 10$  years, and deadweight costs of  $\phi = 0.00$ . The cost function is  $c(M) = 50 \exp(-1.5M)$ . The bottom panel shows the negative of the cost function to demonstrate the  $M$  at which the marginal spread curve intersects the marginal restructuring cost curve.